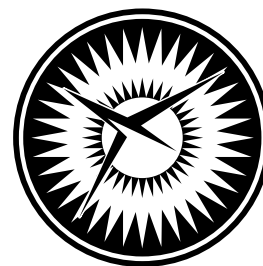


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# Modeling of Unit-Cells with Z-Pins Using FLASH: Pre-Processing and Post-Processing

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# Modeling of Unit-Cells with Z-Pins Using FLASH: Pre-Processing and Post-Processing

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# MODELING OF UNIT-CELLS WITH Z-PINS USING *FLASH*: PRE-PROCESSING AND POST-PROCESSING

Ronald Krueger<sup>1</sup>

## ABSTRACT

Although the toughening properties of stitches, z-pins and similar structures have been studied extensively, investigations on the effect of z-pins on the in-plane properties of laminates are limited. A brief summary on the effect of z-pins on the in-plane tensile and compressive properties of composite laminates is presented together with a concise introduction into the finite element code *FLASH*. The remainder of the report illustrates the modeling aspect of unit cells with z-pins in *FLASH* and focuses on input and output data as well as post-processing of results.

## LIST OF SYMBOLS

|                      |   |
|----------------------|---|
| $C$                  | length of resin pocket parallel to fiber direction            |
| $A_r$                | fraction of the total reinforced area covered by z-pins       |
| $A_z$                | cross sectional area of a single z-pin                        |
| $D_z$                | z-pin diameter  |
| $D_z'$               | height of z-pin plus resin pocket normal to fiber direction   |
| $d$                  | fiber diameter  |
| $E_{11}, E_L$        | Stiffness of lamina parallel to fiber direction               |
| $E_{22}, E_T$        | Stiffness of lamina transverse to fiber direction             |
| $E_{Tc}$             | Compression stiffness of lamina transverse to fiber direction |
| $G$                  | Shear modulus   |
| $G_f$                | Fiber shear modulus   |
| $G_{LT}, G_{12}$     | Lamina shear modulus in principal material directions         |
| $G_{sec}$            | Lamina secant shear modulus                                   |
| $H_z$                | vertical spacing between z-pins in unit cell                  |
| $L_z$                | Horizontal spacing between z-pins in unit cell                |
| $n$                  | Ramberg-Osgood curve fitting parameter                        |
| $N_x, N_y$           | Axial force resultant on laminate in X,Y direction            |
| $N_{xy}$             | Shear force resultant on laminate in X-Y plane                |
| $r_z$                | areal density of z-pins                                       |
| $u_x, u_y$           | Displacement in X, Y direction                                |
| $V_f$                | Fiber volume fraction   |
| $w$                  | kink band width   |
| $\alpha$             | Ramberg-Osgood curve fitting parameter                        |
| $\beta$              | kink band inclination angle                                   |
| $\Delta x, \Delta y$ | distance of fiber misalignment data points in X,Y direction   |
| $\gamma_e$           | Effective shear strain  |
| $\gamma_s$           | shear strain  |

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|                            |  |
|----------------------------|--|
| $\gamma_p$                 | shear strain   |
| $\gamma_y$                 | Yield strain in shear                                  |
| $\lambda$                  | loading parameter                                      |
| $\nu_{12}$                 | Lamina Poisson's ratio                                 |
| $\bar{\phi}$               | Fiber misalignment angle                               |
| $\sigma_c$                 | Strength, critical value of stress                     |
| $\sigma_{Ty}$              | Yield strength in tension                              |
| $\sigma_{xx}, \sigma_{yy}$ | Normal stress in X, Y direction                        |
| $\sigma_{xy}, \sigma_{yx}$ | Shear stress in X-Y plane                              |
| $\sigma_{ult}$             | Strength of skin/stiffener-flange laminate             |
| $\sigma_{ultc}$            | Compression strength of skin/stiffener-flange laminate |
| $\sigma_{11}, \sigma_{22}$ | Normal stress in 1, 2 direction                        |
| $\sigma_{12}, \sigma_{21}$ | Shear stress in 1-2 plane                              |
| $\tau_e$                   | Effective yield strength in shear                      |
| $\tau_y$                   | Yield strength in shear                                |
| $\tau_{xy}, \tau_{yx}$     | Shear stress in X-Y plane                              |
| $\tau_{12}, \tau_{21}$     | Shear stress in 1-2 plane                              |

## 1. BACKGROUND

One of the most common failure modes for composite structures is delamination [1-3]. The remote loadings applied to composite components are typically resolved into interlaminar tension and shear stresses at discontinuities that create delaminations under mixed-mode conditions. In the past research has focussed on stitching to increase the delamination resistance of composites [4-8]. More recently z-pins<sup>2</sup> have been proposed to stitch the material together through a combination of friction and adhesion [9-12]. Z-pins are pultruded rods of carbon fiber and epoxy matrix with an approximate diameter of 280  $\mu\text{m}$  or 508  $\mu\text{m}$ . The z-pins are inserted through the thickness of a laminated composite, which is then autoclaved to cure the epoxy as normal. This approach to through-thickness reinforcement offers an alternative to stitching, and can provide much higher areal densities of reinforcement [13].

Examination of the literature shows that the toughening properties of stitches, z-pins and similar structures have been studied extensively, however, investigations on the effect of z-pins on the in-plane properties of laminates are limited [13-17]. Steeves examined the effect of z-pins on the in-plane tensile and compressive properties of composite laminates [13]. Disruption in the alignment of the fibers in a fiber composite leads to a significant reduction in the in-plane compressive strength [18]. Since the diameter of the z-pins ( $\sim 280 \mu\text{m}$ ) is large relative to the diameter of the fibers ( $\sim 7 \mu\text{m}$ ) in the composite, the z-pins may cause significant misalignment of the fibers of the composite. A sketch of the typical distribution of fiber misalignment around a z-pin is shown in Figure 1. The z-pin is surrounded by a resin-rich pocket on two sides, caused by the fibers deflecting around the z-pin and leaving a gap in the composite. The maximum misalignment of the deflecting fibers occurs in four locations on the four flanks of the resin-rich pocket.

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<sup>2</sup> The generic term z-pin will be used throughout the paper versus the trade mark **Z-Fiber™** registered by Aztex Inc.

Steeves' experiments and numerical finite element simulations using *FLASH* showed that z-pins reduce the in-plane strength of composite laminates. His experimental data from compressive tests showed that the insertion of a field of z-pins into a laminated fiber composite reduces the in-plane compressive strength of the composite by 30% or more. He assumed that the reduction in compressive strength is a consequence of the increased fiber waviness caused by the z-pins [13]. Compression tests within the scanning electron microscope showed that the z-pinned specimens develop microbuckles in the region of greatest fiber waviness around the resin-rich pocket, as would be predicted by the microbuckling model of Budiansky and Fleck [19].

The objective of this report is to present a brief introduction into the finite element code *FLASH* and illustrate the modeling aspect of unit cells with z-pins in *FLASH*. The report focuses on the detailed description of input data for *FLASH* and output obtained from the analysis as well as the post-processing of results. The report is intended to supplement existing user manuals [20, 21]. First, input is described for a finite element model and material data from reference [13]. Second, the correct load input and boundary conditions for compression, tension, shear and combined loading is illustrated. Third, for a composite made of carbon/epoxy modeling for large (0.508 mm diameter) and small (0.28 mm diameter) z-pins is described for 2% and 4% z-pin areal density. Input data are determined for unit cells subjected to five different load cases such as axial compression, combined axial compression and transverse tension, combined axial compression and shear loading as well as combined axial compression, transverse tension and shear loading. Additionally, the output obtained from *FLASH* is explained and separate post-processing options are discussed. The interpretation of the output with respect to the strength of z-pin reinforced lamina is discussed in detail in a separate report [22].

## 2. INTRODUCTION TO *FLASH*

In order to better assess the influence of critical parameters on lamina compression strength, Fleck and Shu developed a finite element code called *FLASH* [20, 21, 23]. The FE code is based on a 2D general Cosserat couple stress theory that assumes the unidirectional composite lamina is a homogeneous anisotropic material that carries couple stress as well as classical Cauchy point stress [24, 25]. The constitutive response is deduced from a unit cell consisting of a fiber, represented by a linear elastic Timoshenko beam, embedded in a non-linear elastic-plastic matrix [26]. The fiber diameter,  $d$ , is the length scale in the constitutive law that controls fiber bending resistance. The continuum theory was implemented within a two-dimensional finite element code that uses 6-noded triangular elements with 3 degrees of freedom at each node (two-displacements and one rotation corresponding to rotation of the fiber cross section). The finite element procedure is based upon a Lagrangian formulation of the finite deformation of the composite and can accommodate both geometric and material non-linearities [23]. The code models finite deformation using a Newton-Raphson incremental solution procedure with a modified Riks algorithm in the final stage to handle snap-back behavior associated with fiber micro-buckling [27]. Boundary loading is piecewise proportional with a loading parameter,  $\lambda$ , for each loading stage [20, 21, 23].

The *FLASH* code assumes micro-buckling initiates from an imperfection in the form of fiber waviness. Inputs include material properties (volume fraction  $V_f$ , and stiffness properties

$E_L, E_T, E_{Tc}, G_{LT}, G_f$ , all normalized by the shear yield strength,  $\tau_y$ ) and the Ramberg-Osgood strain hardening law parameters ( $\alpha, n$ ). *FLASH* allows options for input of fiber misalignment angle due to fiber waviness either as (1) an elliptical patch of waviness, or (2) an arbitrary distribution of initial fiber waviness through initial misalignment angle,  $\bar{\phi}$ , at the Gauss integration point for each element [20, 21, 23]. The first option prescribes the elliptical patch along one edge of the unit cell. Steeves used the second option to input fiber misalignment distribution obtained from analysis of digital images of specimens taken in a Scanning Electron Microscope (SEM) [13].

### 3. GENERIC PRE-PROCESSING

#### 3.1 Unit Cell Geometry

The z-pin reinforced composite is divided into unit cells as shown in Figure 2. The size of the unit cell depends on the areal density,  $r_z$  (in %) of the z-pins and the diameter of a single z-pin,  $D_z$  as shown in Figure 2. The spacing  $L_z=H_z$  for a perfect, rectangular z-pin field can be calculated as

$$L_z = H_z = \sqrt{\frac{A_z}{A_r}},$$

where  $A_z$  is the cross sectional area of a single z-pin

$$A_z = \frac{\pi D_z^2}{4}$$

and  $A_r$  denotes the fraction of the total reinforced area covered by z-pins

$$A_r = \frac{r_z}{100}.$$

The length of the resin pocket,  $C$ , may be determined from micrographs of the reinforced laminate as shown in Figure 3.

#### 3.2 Input of Geometrical Points

For the analysis with *FLASH* only one unit cell is modeled as shown in Figure 4. The resin-rich pocket and the z-pin are modeled as voids within the material. Considering axial compression (x-direction) as the driving failure load, modeling a void is justified because the matrix strength and stiffness are low relative to the strength and stiffness of the composite, and the resin-rich pocket can be considered to be significantly cracked early in the simulation. Hence, the simulation models the unit cell of a composite including fiber misalignment and internal holes [13]. All meshes generated were composed of six-noded triangular plane-strain elements. The size of the elements



was varied to provide the greatest mesh refinement near the resin pocket, and in the region of greatest fiber misalignment.

First geometric parameters were taken from reference [13]. Since the fiber is completely surrounded by resin the transverse dimension of the modeled void,  $D'_z$ , is increased by 0.02 mm compared to the z-pin diameter,  $D_z$  as shown in Figure 3a. The finite element code *FLASH* requires the input of geometric data normalized with the fiber diameter of the composite,  $d$  as listed in Table 1. Geometrical data for the areas to be meshed is input using the **\*GEOMETRICAL POINTS** command in *FLASH*. The coordinates of the points used to create the finite element model of the unit cell with z-pin are printed below and their physical location is sketched in Figure 5.

| <b>*GEOMETRICAL POINTS</b> |        |        |
|----------------------------|--------|--------|
| 26                         |        |        |
| 1                          | 0.00   | 0.00   |
| 2                          | 125.00 | 0.00   |
| 3                          | 250.00 | 0.00   |
| 4                          | 0.00   | 103.60 |
| 5                          | 125.00 | 103.60 |
| 6                          | 250.00 | 103.60 |
| 7                          | 110.00 | 105.00 |
| 8                          | 140.00 | 105.00 |
| 9                          | 0.00   | 124.00 |
| 10                         | 33.00  | 124.00 |
| 11                         | 217.00 | 124.00 |
| 12                         | 250.00 | 124.00 |
| 13                         | 32.00  | 125.00 |
| 14                         | 217.99 | 125.00 |
| 15                         | 0.00   | 126.00 |
| 16                         | 33.00  | 126.00 |
| 17                         | 217.00 | 126.00 |
| 18                         | 250.00 | 126.00 |
| 19                         | 110.00 | 145.00 |
| 20                         | 140.00 | 145.00 |
| 21                         | 0.00   | 146.40 |
| 22                         | 125.00 | 146.40 |
| 23                         | 250.00 | 146.40 |
| 24                         | 0.00   | 250.00 |
| 25                         | 125.00 | 250.00 |
| 26                         | 250.00 | 250.00 |

The geometric parameters were used to generate the finite element mesh of the unit cell including the resin-rich pocket as shown in Figure 4.

### 3.3 Input of Mesh Data

Mesh seed data for the areas to be meshed is input using the **\*MESH SETS** command in *FLASH*. The input to create 10 mesh sets which form the finite element model of the unit cell with z-pin (see Figure 4) are printed below. The physical location of the mesh sets is sketched in Figure 6.

```
*MESH SETS
10,6
1, 1, 4, 2, 2, 5, 8, 10, 0.1, 0.1, 0.1, 0.1
2, 2, 5, 3, 3, 6, 8, 10, 0.1, 0.1, 10, 10
4, 4, 9, 5, 7, 10, 8, 10, .25, .25, 0.1, 0.1
5, 8, 11, 6, 6, 12, 8, 10, .25, .25, 10, 10
9, 9, 15, 10, 13, 16, 3, 10, 1, 1, 0.1, 0.1
11, 14, 17, 12, 12, 18, 3, 10, 1, 1, 10, 10
15, 15, 21, 16, 19, 22, 8, 10, 4, 4, 0.1, 0.1
17, 20, 22, 18, 18, 23, 8, 10, 4, 4, 10, 10
21, 21, 24, 22, 22, 25, 8, 10, 10, 10, 0.1, 0.1
22, 22, 25, 23, 23, 26, 8, 10, 10, 10, 10, 10
```

### 3.4 Input of Misalignment Data

The finite element code *FLASH* also calls for the input of the fiber misalignment angle [20, 21]. The misalignment angle is input using the **\*MISALIGNMENT TYPE** and the **\*MISALIGNMENT** command in *FLASH*. The input used for all analysis is printed below.

```
*MISALIGNMENT TYPE
2
*MISALIGNMENT
74,74,3.425,3.425
```

Preliminary analysis with  $N=1$  as input for **\*MISALIGNMENT TYPE** which was supposed to generate an elliptical patch of waviness was not successful. Post-processing – discussed later - only showed a small band of waviness along the left edge of the specimen ( $x=0$ ). The waviness in the rest of the specimen was zero. Therefore this approach was abandoned and  $N=2$  was chosen which requires that the discretized misalignment data is provided in a separate input file (**phiex.dat**). Input of an arbitrary distribution of the fiber misalignment is possible as by sampling a micrograph as shown in Figure 7 [13]. However, if this information is not available, misalignment angles may be chosen to study the misalignment effect on the reduction of in-plane strength.

To illustrate the input an example is shown in Figure 8 for 10 by 10 sample points on the unit cell which corresponds to a matrix of 10 by 10 data points in the **phiex.dat** file. The distance of one point to the adjacent points was set to  $\Delta x=26.0$  units in  $x$  and  $\Delta y=26.0$  units in  $y$ .

direction which extends beyond the unit cell (250 by 250 units) modeled. The distance of points has to be chosen so that the field of data points is larger than the unit cell. The user is advised to check the misalignment input after the execution of **prep** has generated the **flash.inp** file and adjust the distance of adjacent points accordingly. If the misalignment data originates from sampling a micrograph the sample area has to be larger than the model. See reference [21] for details. Only the file named **phiex.dat** will be used during the initial step of the analysis when the file **flash.inp** is generated during the execution of **prep**.

For the current analyses constant fiber misalignment values of  $0^\circ$ ,  $1^\circ$ ,  $2^\circ$ ,  $3^\circ$ ,  $4^\circ$ ,  $5^\circ$ ,  $6^\circ$ ,  $7^\circ$ ,  $8^\circ$ ,  $9^\circ$ , and  $10^\circ$ , were chosen and 11 **phiex.dat** files were created. Each **phiex.dat** file contains a field of 74 by 74 data points which corresponds to a matrix of 74 by 74 sample points on the unit cell as shown for the example of  $5^\circ$  fiber misalignment.

```

5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000
5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000
5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000
5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000
5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000
5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000 5.000
5.000 5.000
5.000 5.000 5.000 5.000 5.000 5.000 ...
...

```

The distance of one point to the adjacent points was set to 3.425 units in  $x$  and 3.425 units in  $y$ -direction to cover the entire unit cell (250 by 250 units).

### 3.5. Input of Constraints

As mentioned above the composite is divided into unit cells, and only one cell is modeled using periodic boundary conditions. For these models, periodic boundary conditions are prescribed by applying compressive loading symmetrically to the specimen, and constraining three degrees of freedom to prevent rigid body translation and rotation as shown in Figure 9. The constraints are input using the **\*TYPE 1 CONSTRAINTS** command in *FLASH*. The input used for all analysis is printed below.

```

*TYPE 1 CONSTRAINTS
3
1, 1, 2, 21, 1, 1, 0
9, 9, 15, 4, 4, 2, 0
24, 24, 25, 21, 1, 1, 0

```

### 3.6. Input of Load Data

The loading was input using the **\*STRESS LOADING** command in *FLASH*. The axial compression stress sketched in Figure 9 is gradually incremented by *FLASH* until it reaches the specified limit defined by the user ( $\sigma_{xx}/\tau_y = -1000$ ). The limit was deliberately chosen to be well above failure, i.e. formation of kink bands and fiber microbuckling to assure that the analysis reached the failure point and did not terminate at a chosen lower stress [20]. The input used for axial compression loading is printed below.

```
*STRESS LOADING
10, 1
1, 1, 4, 1, 1
-1000, 0, 0, 0, 0
4, 4, 9, 1, 1
-1000, 0, 0, 0, 0
9, 9, 15, 1, 1
-1000, 0, 0, 0, 0
15, 15, 21, 1, 1
-1000, 0, 0, 0, 0
21, 21, 24, 1, 1
-1000, 0, 0, 0, 0
3, 3, 6, 1, 1
-1000, 0, 0, 0, 0
6, 6, 12, 1, 1
-1000, 0, 0, 0, 0
12, 12, 18, 1, 1
-1000, 0, 0, 0, 0
18, 18, 23, 1, 1
-1000, 0, 0, 0, 0
23, 23, 26, 1, 1
-1000, 0, 0, 0, 0
```

### 3.7. Requests for Output

During the analysis stresses, strains and total fiber alignment angle is written to the file *flash.out* after every  $N$  load steps. The data output request is managed using the **\*OUTPUT FREQUENCY** commands in *FLASH* as printed below for an output request every 1000 steps.

```
*OUTPUT FREQUENCY
1000
```

During the analysis, for three boundaries (as shown in Figure 9) data are output to the output file *post.d* after each load step. For boundary sections 1 and 2 the net force on each boundary section, resolved along the nominal fiber direction, are given as output. Boundary 3 is a

single node and the displacements of this node are given as output. The data output request is managed using the **\*OUTPUT LIST** command in *FLASH* as printed below. The boundary sections selected are shown in Figure 5.

```
*OUTPUT LIST
3
1, 1, 4, 1, 1
1, 1, 4, 1, 1
1, 1, 4, 1, 1
```

Additionally, the value of the global loading parameter  $\lambda$  is written to the output file at each loading step.

The format of the output files and post-processing of the results is discussed in a separate section below.

### 3.8. Input of Material Data

The finite element code *FLASH* requires the input of material data normalized with the composite yield shear strength  $\tau_y$  as given in Table 2 for a generic graphite epoxy material [13]. Additional material parameters used by *FLASH* are the constants  $\alpha=0.429$  and  $n=3$  in the Ramberg-Osgood strain hardening law, and the fiber volume fraction  $V_f=0.63$  of the composite. The material data is input using the **\*MATERIAL CONSTANTS** and **\*VFRACTION** commands in *FLASH* as printed below.

```
*MATERIAL CONSTANTS
7
1200, 175, 74, 41, 0.429, 3.0, 87
*VFRACTION
0.63
```

### 3.9. Input for a Finite Element Model without Z-Pin

For reference purposes a unit cell without z-pin was modeled and analyzed with *FLASH*. The mesh with applied load and boundary condition is shown in Figure 10. The geometrical points and the required input data to generate the mesh have been included in Figure 10.

## 4. PREPROCESSING FOR COMBINED LOADING

For the initial analyses performed, load and boundary conditions were taken from reference [13]. Compressive loading was applied symmetrically to the specimen, and three degrees of freedom were constrained to prevent rigid body translation and rotation as shown in Figure 9.

In general, z-pin reinforced composite structures are subjected to multi axial loads and

input stresses in the unidirectional ply modeled in *FLASH* need to be calculated. For composite plates subjected to external compression and combined compression and shear loading input stresses for *FLASH* were calculated. Using classical laminate theory the external loads were resolved into stresses for the individual plies as shown in Figure 11. Calculated stresses in the critical unidirectional ply consist of axial compression, transverse compression and tension as well as shear stresses. For these combined loading conditions the above mentioned boundary conditions and load input had to be modified.

#### **4.1. FE Models of Z-Pin Unit Cells Subjected to Axial and Transverse Loading**

The finite element mesh with load and boundary conditions for axial and transverse compression is shown in Figure 12 with the corresponding input data. Based on results from classical laminate theory mentioned above, the transverse compression load was chosen to be 2% of the compression load applied axially. In order to avoid the collapse of the void under transverse compression points A and B on the periphery of the void could be constrained in transverse direction as shown in Figure 12. In reality, the boundary condition of zero displacements on the periphery of the void is not met due to pin compliance. However, the assumed boundary condition is adopted in order to obtain an estimate for the extreme case of a rigid pin [28]. Nevertheless, analysis without constraints suggested that constraining points A and B for small transverse compression loads leads to overly conservative results.

The finite element mesh with load and boundary conditions for axial compression and transverse tension are shown in Figure 13. As for the previous case the transverse load was chosen to be 2% of the compression load applied axially. For transverse tension it is not required to avoid the collapse of the void.

#### **4.2. FE Models of Z-Pin Unit Cells Subjected to Axial Compression and Shear Loading**

The finite element mesh with load and boundary conditions for axial compression combined with shear are shown in Figure 14. Based on results from classical laminate theory mentioned above, the shear load was chosen to be 10% of the compression load applied axially. For the combined axial compression and shear load case it was decided not to prevent the collapse of the void as explained in detail in the appendix of reference 23.

The finite element mesh with load and boundary conditions for axial compression and transverse tension combined with shear are shown in Figure 15. As for the previous case the shear load was chosen to be 10% of the compression load applied axially and the transverse tension load corresponded to 2% of the compression load as discussed in the previous section. For this combined load case it was decided not to prevent the collapse of the void as explained in detail in the appendix of reference 23.

### **5. UNIT CELL MODELS FOR 2% AND 5% Z-PIN REINFORCEMENT.**

For another carbon fiber reinforced epoxy material the data used as input for the analyses with *FLASH* are listed in Table 3. The specimens were reinforced with small z-pins of

$D_z=0.279$  mm (0.011 in) and large z-pins of  $D_z=0.508$  mm (0.02 in) diameter. Three specimen types were manufactured containing reinforcement fields with  $r_z=2\%$  areal density for the large z-pin and  $r_z=2\%$  and  $r_z=4\%$  areal density for the small z-pins respectively. Geometric data such as z-pin spacing,  $L_z=H_z$ , as well as width,  $D'_z$ , and length,  $C$ , of the resin pocket as defined in Figure 3 was measured from micrographs taken of different specimens. The averaged data used as input for the analysis is summarized in Table 4.

Based on results from classical laminate theory shown in Figure 11 and Table 5, five load cases were considered as input for analysis of the unit cells: A pure axial compression load case, a combined axial compression - 2% transverse tension load case, a combined axial compression - 10% shear load case, a combined axial compression - 50% shear load case, and an axial compression - 2% transverse tension load case combined with 10% shear loading. The input for all five cases is summarized in Table 6.

The finite element meshes are shown in Figures 16, 17, and 18 for the small pin with 2% and 4% areal density and the large pin with 2% areal density. As before the resin-rich pocket and the z-pin are modeled as voids within the material. It is assumed that the fiber is completely surrounded by resin as shown in Figure 3 and therefore the transverse dimension of the modeled void,  $D'_z$ , is increased by 0.02 mm compared to the z-pin diameter,  $D_z$ . The normalized dimensions required as input were included in the figures as well as the input for the **\*GEOMETRICAL POINTS** command in *FLASH*. Loads given in Table 6 were applied as discussed in section 4.

## 6. POSTPROCESSING OF RESULTS

### 6.1. FLASH Output Files

During the analysis nodal point coordinates, element topology and results are is written to the file **flash.out** after every  $N$  load steps. The file **flash.out** contains a header,

```
1Ccomnew 0.00000E+00          FIBER MICROBUCKLING  3
```

the nodal point coordinates

```
2C
-1    1 0.00000E+00 0.00000E+00
-1    2 0.00000E+00 0.15646E+02
-1    3 0.00000E+00 0.31293E+02
-1    4 0.00000E+00 0.42553E+02
-1 ...
-3
```

the element topology

```
3C
-1    1      8
-2    1  374  372  155  373  154
```

```

-1      2      8
-2      1      3 374      2 156 155
-1      3      8
-2 372 710 708 581 709 580
-1 ...
-3

```

the initial fiber misalignment  $\bar{\phi}$

```

-3
 100CLCASE1 0.00000E+00      Fiber Misalignment  3      0
-4 MISALIGN      1      3      0
-5 PHIBAR      1      1      1      0      0
-1      1      8      0      0      6
-2      1 0.40000E+01
-2      2 0.40000E+01
-2      3 0.40000E+01
-2      4 0.40000E+01
-2      5 0.40000E+01
-2      6 0.40000E+01
-1      2      8      0      0      6
-2      1 0.40000E+01
-2      2 0.40000E+01
-2      3 0.40000E+01
-2      4 0.40000E+01
-2 ...
-3

```

the normalized displacements  $u_x/d$  and  $u_y/d$  at all nodal points

```

100CLCASE1 0.12052E-01      Fiber Microbuckling  3 1000
-4 DISPLACE      3      1      0
-5 UX      1      2      1      0      0
-5 UY      1      2      2      0      0
-5 U      1      2      0      0      1ALL
-1      1 0.14712E+01 -0.32663E+01
-1      2 0.80530E+00 -0.32449E+01
-1      3 0.14974E+00 -0.31895E+01
-1      4 -0.31371E+00 -0.31129E+01
-1 ...
-3

```

the normalized stresses  $\sigma_{xx}/\tau_y$ ,  $\sigma_{yy}/\tau_y$ ,  $\sigma_{xy}/\tau_y$  and  $\sigma_{yx}/\tau_y$ , the normalized effective shear stress



$\tau_e/\tau_y$ , the strains  $\gamma_s$  and  $\gamma_p$ , the fiber misalignment  $\bar{\phi}$  and others [20]

```

100CLCASE1 0.12052E-01          Fiber Microbuckling 3 1000
-4 STRSSTRN 12 3 0
-5 SIG_XX 1 4 1 1 0
-5 SIG_YY 1 4 2 2 0
-5 SIG_XY 1 4 1 2 0
-5 SIG_YX 1 4 2 1 0
-5 TAU_E 1 1 2 1 0
-5 E_LP 1 1 2 1 0
-5 E_LM 1 1 2 1 0
-5 E_T 1 1 2 1 0
-5 GAMA_S 1 1 2 1 0
-5 KAPPA 1 1 2 1 0
-5 GAMA_P 1 1 2 1 0
-5 PHI 1 1 2 1 0
-1 1 8 0 0 6
-2 1 -0.12328E+02 0.20839E+00 -0.11766E+00 -0.13105E+00 0.14941E+01 -0.30784E-02
-2 1 -0.25050E-02 0.17207E-03 0.20919E-01 -0.57344E-03 0.10263E-01 0.76157E+01
-2 2 -0.11658E+02 0.44093E+00 -0.74204E+00 -0.73252E+00 0.74208E+00 -0.28156E-02
-2 2 -0.25441E-02 0.18100E-02 0.60815E-02 -0.27141E-03 0.13715E-02 0.66904E+01
-2 3 -0.13010E+02 0.17288E-01 -0.74737E+00 -0.74508E+00 0.79505E+00 -0.30924E-02
-2 3 -0.28856E-02 0.63897E-04 0.72372E-02 -0.20685E-03 0.15640E-02 0.67907E+01
-2 4 -0.11966E+02 0.32614E+00 -0.32097E+00 -0.30034E+00 0.12303E+01 -0.29931E-02
-2 4 -0.24544E-02 0.11352E-02 0.14406E-01 -0.53866E-03 0.57257E-02 0.72148E+01
-2 ...
-3

```

During the analysis, data for three boundaries are output to the output file **post.d**. The file **post.d** contains seven columns. Column 1 lists the load step number, column 2 lists load increase (+1) or decrease (-1), columns 3 and 4 list the normalized displacements  $u_x/d$  and  $u_y/d$  for the selected node on boundary 3, columns 5 and 6 list the net loads on boundary sections 1 and 2 as shown in Figure 9. Column 7 lists global loading parameter  $\lambda$  for each load step, where  $\lambda$  is a measure of how close the simulation is to fulfilling the requested stress loading discussed in section 3.6.

```

0 1 0.00000E+00 0.00000E+00 0.00000E+00 0.00000E+00 0.00000E+00
1 1 -0.24101E-01 -0.13840E-01 0.10106E+03 0.10106E+03 0.96809E-03
2 1 -0.37684E-01 -0.22066E-01 0.15836E+03 0.15836E+03 0.15167E-02
3 1 -0.52874E-01 -0.31723E-01 0.22269E+03 0.22269E+03 0.21325E-02
4 1 -0.69664E-01 -0.43129E-01 0.29416E+03 0.29416E+03 0.28163E-02
...

```

|      |   |              |             |             |             |             |
|------|---|--------------|-------------|-------------|-------------|-------------|
| 996  | 1 | -0.49693E+00 | 0.21751E+00 | 0.19896E+04 | 0.19896E+04 | 0.18922E-01 |
| 997  | 1 | -0.49626E+00 | 0.21723E+00 | 0.19925E+04 | 0.19925E+04 | 0.18949E-01 |
| 998  | 1 | -0.49530E+00 | 0.21693E+00 | 0.19966E+04 | 0.19966E+04 | 0.18988E-01 |
| 999  | 1 | -0.49464E+00 | 0.21683E+00 | 0.19995E+04 | 0.19995E+04 | 0.19015E-01 |
| 1000 | 1 | -0.49373E+00 | 0.21680E+00 | 0.20036E+04 | 0.20036E+04 | 0.19053E-01 |

## 6.2. Result Visualization

The output file `flash.out` is formatted to be used with the commercial post-processing software *FEMGV*<sup>3</sup>. A translation routine (`make_post.f`) was developed, which reads `flash.out` and creates output data of the model (`flash_pat.out`) and displacements (`flash.dis`) in neutral file format to be used with *PATRAN*<sup>4</sup>. The routine can also create output of the model and result data (`flash.dat`) to be used with *Tecplot*<sup>5</sup>. The routine can easily be modified to create an output file in a particular format that is compatible with a graphics package of the user's choice.

### 6.2.1 Examples of Deformation Plots

The deformed finite element meshes for the unit cell under axial compression are shown in Figure 19. At the beginning of the loading a symmetric deformation with respect to the two axis of symmetry can be observed as shown in Figure 19a. At the edge where the axial compression stress is applied a non-uniform displacement distribution can be observed which forms an axial contraction. The increased displacement is caused by a reduced stiffness which is the result of modeling the resin pocket and z-pin as a void. At the end of the analysis after passing the failure point and the formation of kink bands the deformation grows to be unsymmetrical and the unit cell becomes increasingly distorted as shown in Figure 19b.

The deformed mesh for axial and transverse compression is shown in Figure 20. The transverse compression load was 2% of the compression load applied axially as shown in Figure 12. Analysis without constraints as shown in Figure 20 suggested that constraining points A and B shown in Figure 12, was not required for small transverse compression loads. Compared to the deformation shown in Figure 19a for the pure compression case the axial displacement shown in Figure 20 is reduced at the top and bottom edges where the transverse compression is applied and the void remains open.

The deformed mesh for axial compression and transverse tension are shown in Figure 21. As for the previous case the transverse load was chosen to be 2% of the compression load applied axially as shown in Figure 13. Compared to the deformation for the pure compression case as shown in Figure 19a the axial displacement shown in Figure 21 has increased at the top and bottom edges where the transverse tension stress is applied; also the void opening has increased.

<sup>3</sup> <http://www.femsys.co.uk/>

<sup>4</sup> [http://www.mssoftware.com/products/quick\\_prod.cfm](http://www.mssoftware.com/products/quick_prod.cfm)

<sup>5</sup> <http://www.tecplot.com/>

### 6.2.2 Examples of Contour Plots

As an example the contour plots of the fiber misalignment  $\bar{\phi}$ , the effective shear stress  $\tau_e$  and the strains  $\gamma_s$  and  $\gamma_p$ , are shown in Figures 24-27 for the unit cell subjected to axial and transverse compression. The contours correspond to the end of the analysis after passing the failure point where the severe color gradients indicate the formation and location of kink bands.

### 6.2.3 Examples of Stress-Displacement Plots to Obtain Compression Strength

As an example typical stress versus displacements plots are shown in Figures 28 and 29. Typically the `post.d` file is read into a graphing program such as KaleidaGraph™ or Excel™. Here the stress is obtained by multiplying the global loading parameter  $\lambda$  from column (#7) of the output file `post.d` with the matrix yield stress  $\tau_y$  from the Ramberg-Osgood strain hardening law that is used as input and the requested stress loading (-1000) described in section 3.6. The compression stress  $\sigma_{xx} = \lambda \tau_y / 1000$  is plotted versus the normalized displacement in x-direction ( $u_x/d$ ) for the selected node on boundary 3 as shown in Figure 9. The normalized displacement ( $u_x/d$ ) is listed in column (#3) of the output file `post.d`. As shown in Figures 28 and 29, the plot typically indicates a clear first maximum. The exact maximum value, which is the predicted compression strength, is best found in the data rather than the plot.

### ACKNOWLEDGEMENTS.

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## LIST OF TABLES

Table 1.

*Dimensions of Unit Cell*

| Graphite/Epoxy UD Prepreg with 2% Z-Pins [13] |                 |                       |
|---|-----------------|-----------------------|
|   | from            | normalized with $d_f$ |
| $D_Z$   | 0.28 mm         |                       |
| $D'_Z$  | 0.3 mm          | 42.8                  |
| $H_Z$   | 1.75 mm         | 250                   |
| $L_Z$   | 1.75 mm         | 250                   |
| $C$   | 1.29 mm         | 184                   |
| $d$   | 7 $\mu\text{m}$ | 1                     |

Table 2.  
*Material Properties for Initial Preliminary Analysis*

| Graphite/Epoxy Prepreg (A) [13]         |                 |                             |
|---|-----------------|-----------------------------|
|   | from            | normalized with $\tau_{xy}$ |
| $E_{11}$                                | 117 GPa         | 1083                        |
| $E_{22}$ (tension)                      | 9.0 GPa         | 83.33                       |
| $E_{22}$ (compression)                  | 9.5 GPa         | 87.96                       |
| $G_{12}$                                | 4.8 GPa         | 44.44                       |
| $G_f$                                   | 22 GPa          | 203.7                       |
| $\tau_y$                                | 108 MPa         | 1                           |
| $d$                                     | 7 $\mu\text{m}$ | -                           |
| $V_f$                                   | 0.55            | -                           |
| $\alpha$                                | 3/7             | -                           |
| $n$                                     | 3               | -                           |
| Generic Graphite/Epoxy Prepreg (B) [13] |                 |                             |
|   |                 | normalized with $\tau_{xy}$ |
| $E_{11}$                                | -               | 1200                        |
| $E_{22}$ (tension)                      | -               | 74                          |
| $E_{22}$ (compression)                  | -               | 87                          |
| $G_{12}$                                | -               | 41                          |
| $G_f$                                   | -               | 175                         |
| $\tau_y$                                | -               | -                           |
| $d$                                     | 7 $\mu\text{m}$ | -                           |
| $V_f$                                   | 0.63            | -                           |
| $\alpha$                                | 0.429           | -                           |
| $n$                                     | 3               | -                           |

Table 3.  
*Carbon/Epoxy Material Properties*

| Graphite/Epoxy UD Prepreg |                   |                             |
|---------------------------|-------------------|-----------------------------|
|                           | from              | normalized with $\tau_{xy}$ |
| $E_{11}$                  | 161 GPa           | 4248                        |
| $E_{22}$ (tension)        | 11.4 GPa          | 301                         |
| $E_{22}$ (compression)    | 12.8 GPa          | 338                         |
| $G_{12}$                  | 5.17 GPa          | 136                         |
| $G_f$                     | 22 GPa            | 580                         |
| $\tau_y$                  | 37 MPa            | 1                           |
| $d$                       | 5.1 $\mu\text{m}$ | -                           |
| $V_f$                     | 0.59              | -                           |
| $\alpha$                  | 0.00923           | -                           |
| $n$                       | 8.54              | -                           |

Table 4.  
*Dimensions of Unit Cells*

| Graphite/Epoxy UD Prepreg with 2% large diameter Z-Pins |           |                     |
|---|-----------|---------------------|
|   | from      | normalized with $d$ |
| $D_Z$   | 0.508 mm  | -                   |
| $D'_Z$  | 0.528 mm  | 103.53              |
| $H_Z$   | 3.175 mm  | 622.55              |
| $L_Z$   | 3.175 mm  | 622.55              |
| $C$   | 2.1844 mm | 428.31              |
| Graphite/Epoxy UD Prepreg with 4% small diameter Z-Pins |           |                     |
|   | from      | normalized with $d$ |
| $D_Z$   | 0.28 mm   | -                   |
| $D'_Z$  | 0.3 mm    | 58.8                |
| $H_Z$   | 1.2446 mm | 244                 |
| $L_Z$   | 1.2446 mm | 244                 |
| $C$   | 0.868 mm  | 170.2               |
| Graphite/Epoxy UD Prepreg with 2% small diameter Z-Pins |           |                     |
|   | from      | normalized with $d$ |
| $D_Z$   | 0.28 mm   | -                   |
| $D'_Z$  | 0.3 mm    | 58.8                |
| $H_Z$   | 1.7526 mm | 343.65              |
| $L_Z$   | 1.7526 mm | 343.65              |
| $C$   | 0.868 mm  | 170.2               |



Table 5.

*Load Cases for Laminate Analysis and Results*

| external load $N_x = -1000$ lbs/in                          |                     |                             |                             |
|---|---------------------|-----------------------------|-----------------------------|
|   | $\sigma_{11}$ , psi | $\sigma_{22}$ , psi         | $\tau_{12}$ , psi           |
| $[0/90]_s$  | -1898               | -29.5 (~2% $\sigma_{11}$ )  | 0                           |
| $[0/\pm 45]_s$  | -2304               | +48.9 (~2% $\sigma_{11}$ )  | 0                           |
| $[0/45/-45/90]_s$   | -2597               | -3.3 (~0.2% $\sigma_{11}$ ) | 0                           |
| external load $N_x = -1000$ lbs/in, $N_{xy} = -1000$ lbs/in |                     |                             |                             |
|   | $\sigma_{11}$ , psi | $\sigma_{22}$ , psi         | $\tau_{12}$ , psi           |
| $[0/90]_s$  | -1898               | -29.5                       | -1000 (~50% $\sigma_{11}$ ) |
| $[0/\pm 45]_s$  | -2304               | +48.9 (~2% $\sigma_{11}$ )  | -222 (~10% $\sigma_{11}$ )  |
| $[0/45/-45/90]_s$   | -2597               | -3.3                        | -275 (~10% $\sigma_{11}$ )  |

Table 6.

*FLASH Input for Load Cases Used for Strength Reduction Analysis*

|                      | axial<br>compression | compression/<br>2%transverse<br>tension | compression<br>10% shear | compression<br>50% shear | compression<br>2% tension<br>10% shear |
|----------------------|----------------------|---|--------------------------|--------------------------|--|
| $\sigma_{11}/\tau_y$ | -1000                | -1000                                   | -1000                    | -1000                    | -1000                                  |
| $\sigma_{22}/\tau_y$ | -                    | +20                                     | -                        | -                        | +20                                    |
| $\tau_{12}/\tau_y$   | -                    | -                                       | 100                      | 500                      | 100                                    |
| $\tau_{21}/\tau_y$   | -                    | -                                       | 100                      | 500                      | 100                                    |

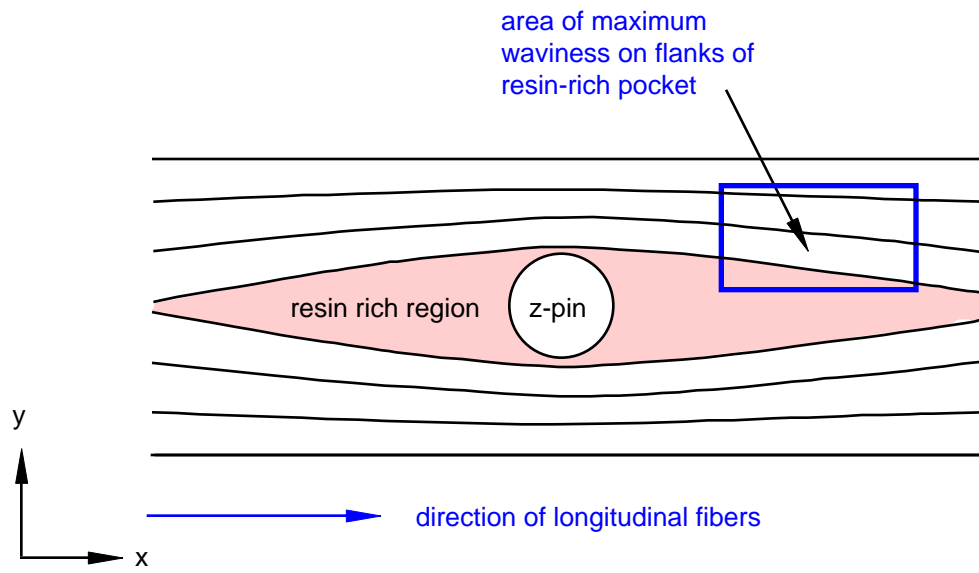


Figure 1: Typical shape of local area around z-pin

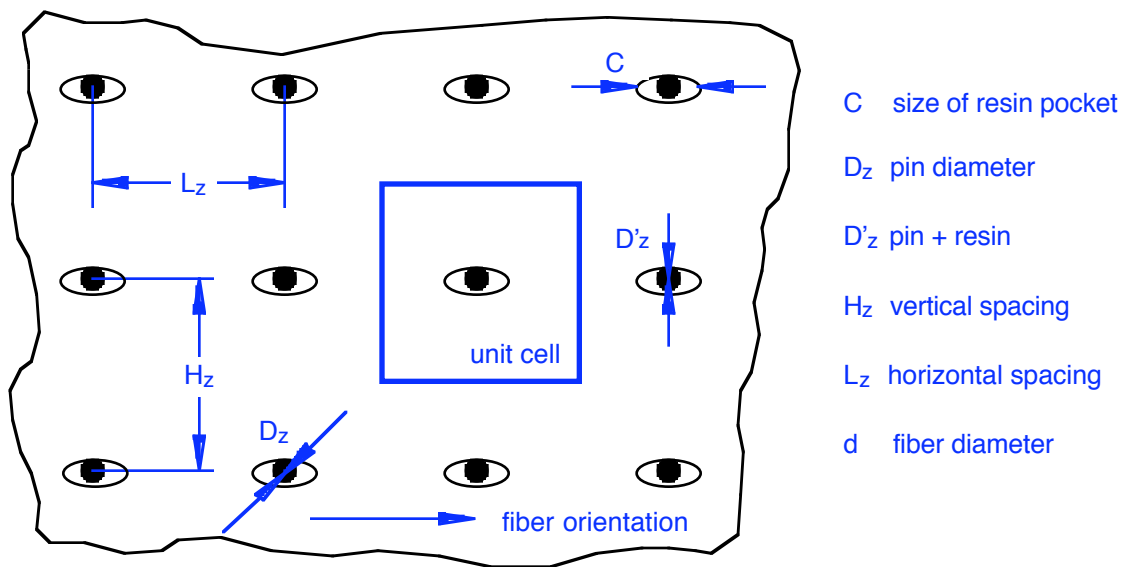


Figure 2: Detail of perfect z-pin field with typical dimensions and unit cell

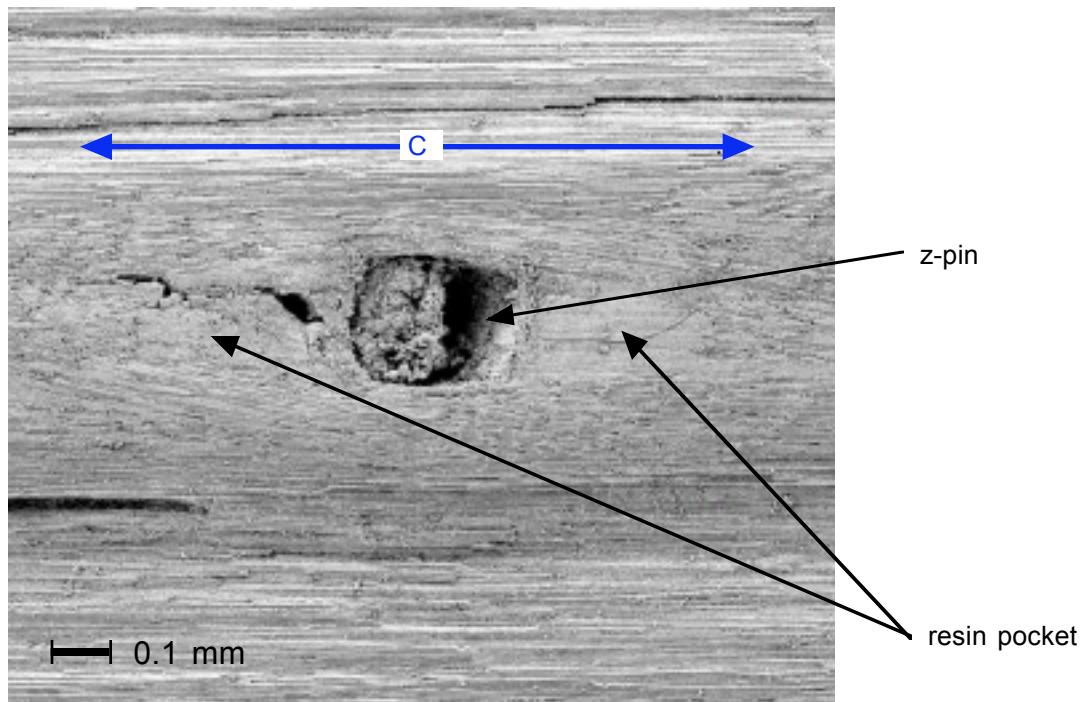
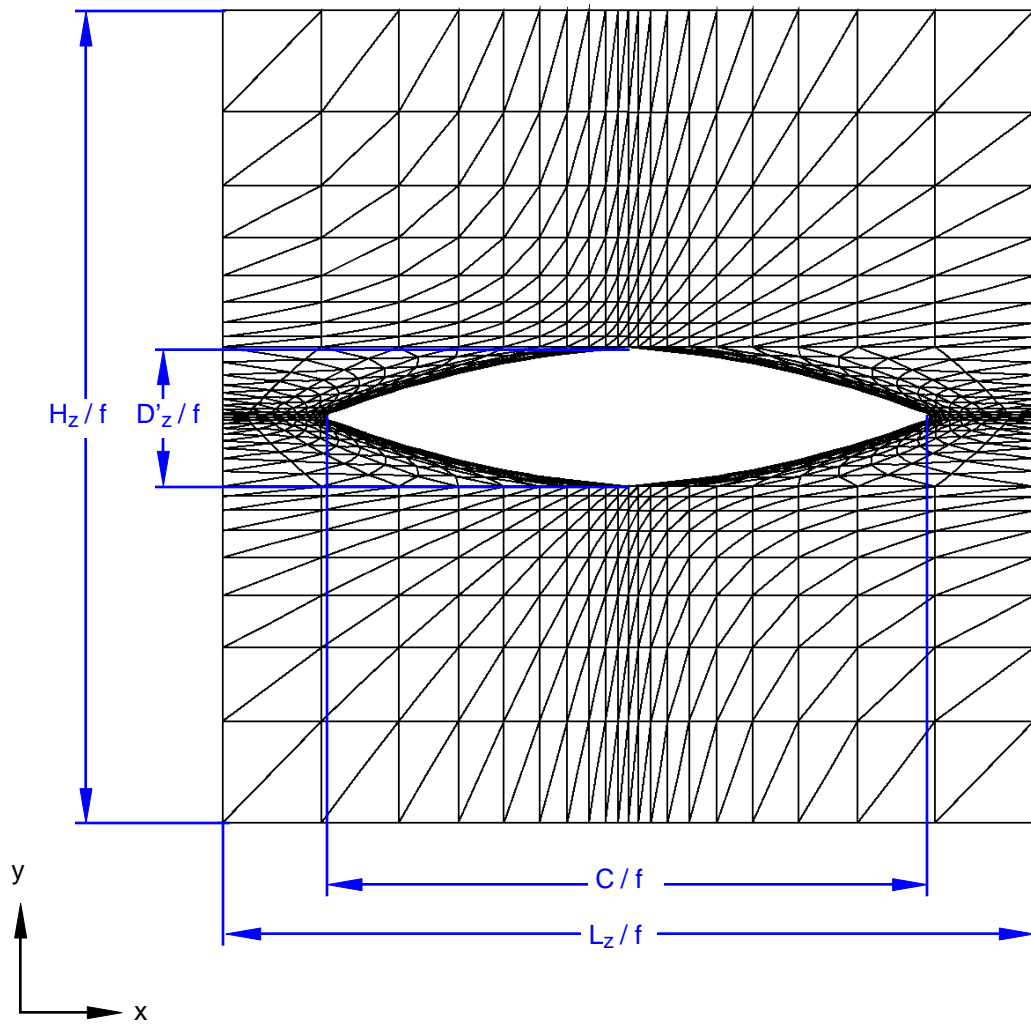


Figure 3: Micrograph of individual z-pin with surrounding laminate [13]



**Figure 4: Typical finite element mesh of unit cell with normalized dimensions and initial misalignment due to z-pin insertion**

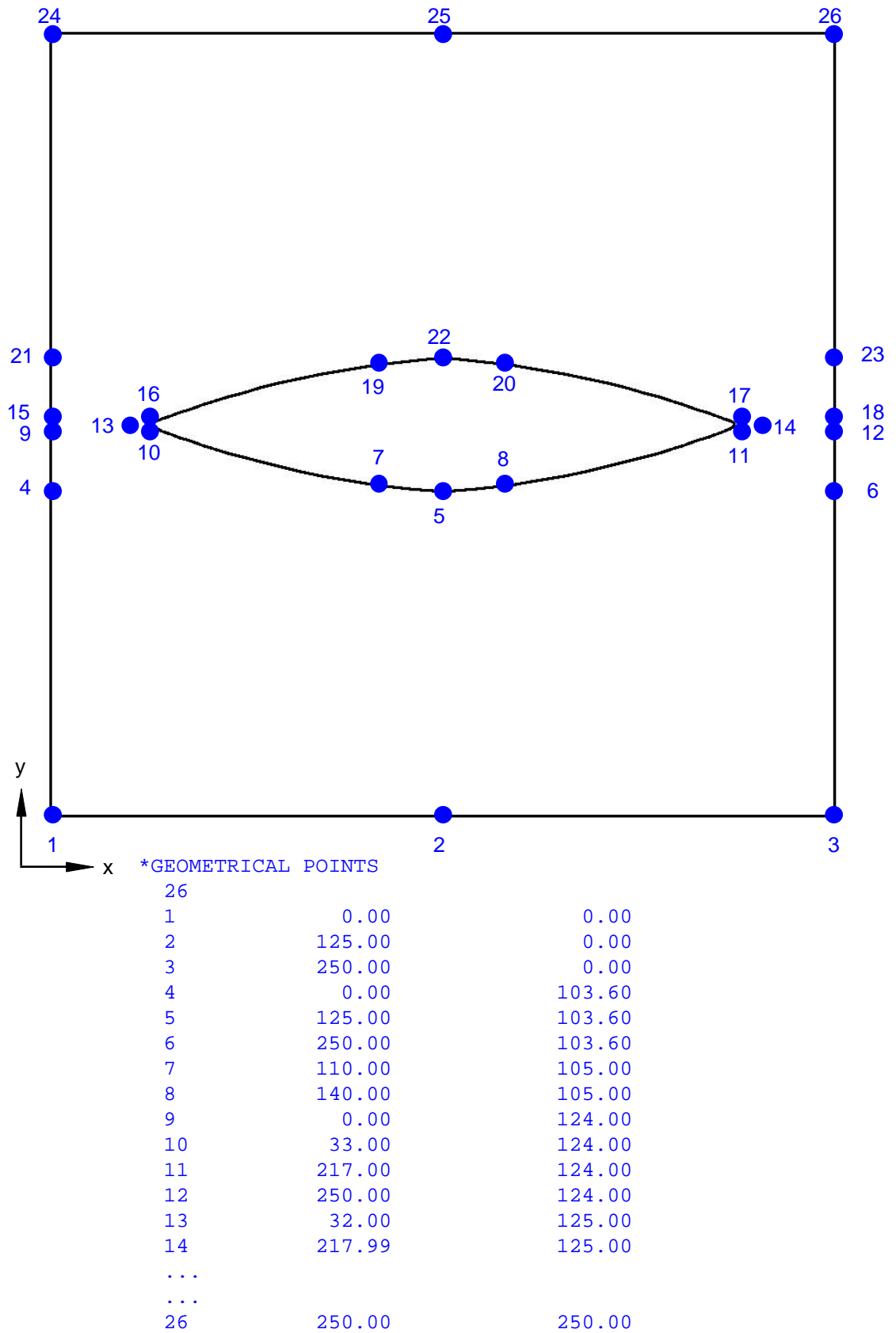
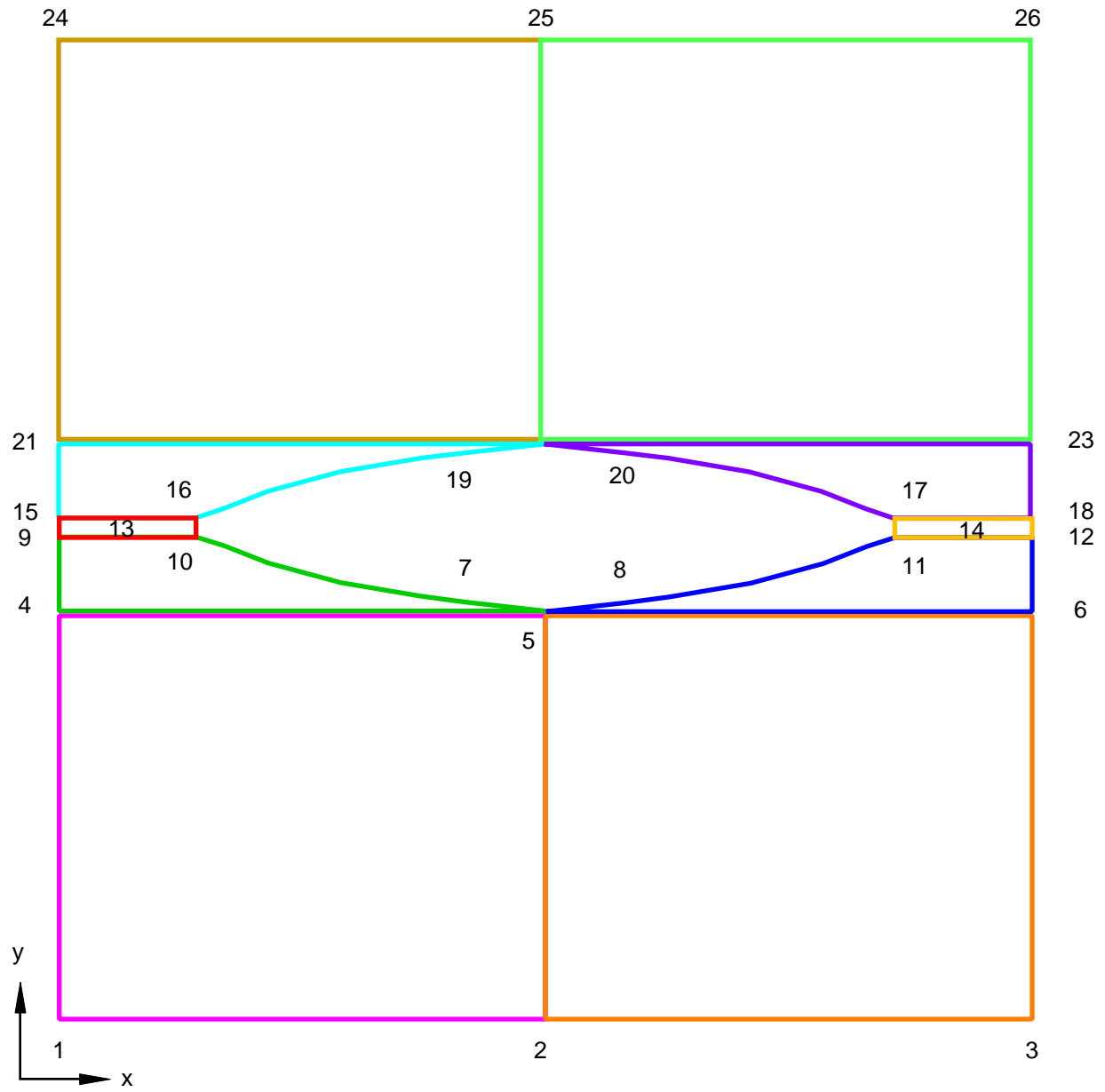


Figure 5: Outline of unit cell and void with geometry points



\*MESH SETS

10,6

1, 1, 4, 2, 2, 5, 8, 10, 0.1, 0.1, 0.1, 0.1

2, 2, 5, 3, 3, 6, 8, 10, 0.1, 0.1, 10, 10

4, 4, 9, 5, 7, 10, 8, 10, .25, .25, 0.1, 0.1

5, 8, 11, 6, 6, 12, 8, 10, .25, .25, 10, 10

9, 9, 15, 10, 13, 16, 3, 10, 1, 1, 0.1, 0.1

11, 14, 17, 12, 12, 18, 3, 10, 1, 1, 10, 10

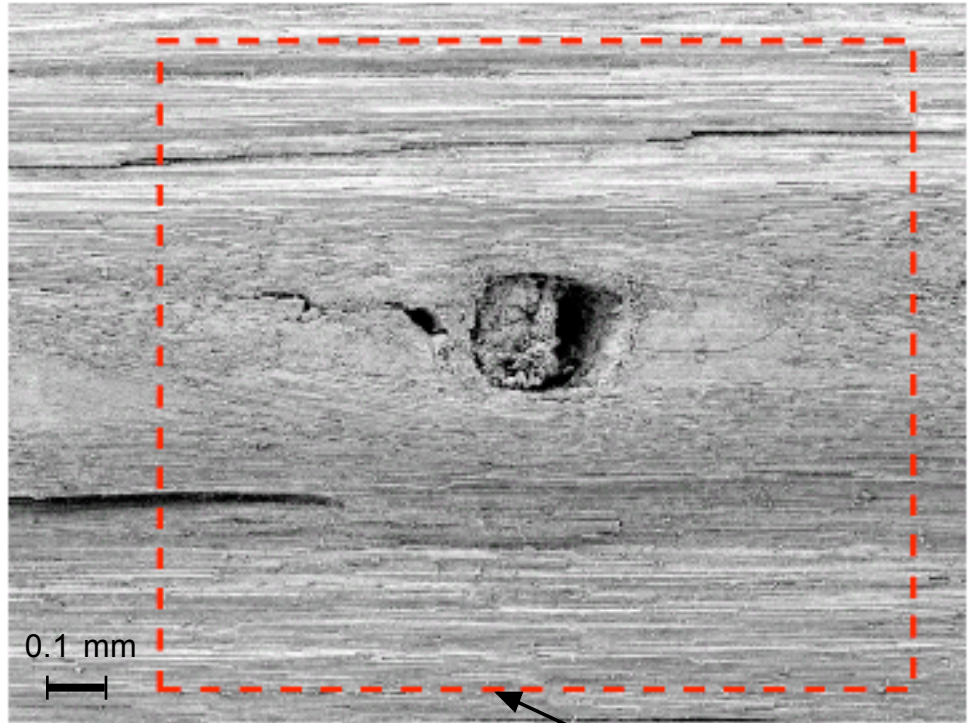
15, 15, 21, 16, 19, 22, 8, 10, 4, 4, 0.1, 0.1

17, 20, 22, 18, 18, 23, 8, 10, 4, 4, 10, 10

21, 21, 24, 22, 22, 25, 8, 10, 10, 10, 0.1, 0.1

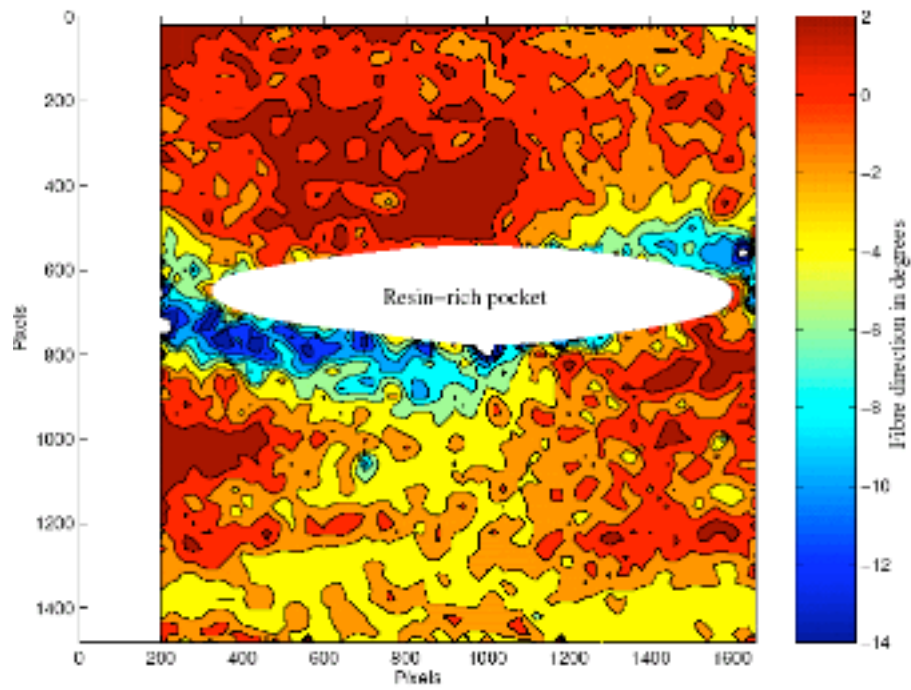
22, 22, 25, 23, 23, 26, 8, 10, 10, 10, 10, 10

Figure 6: Outline of unit cell and void with geometry points and mesh sets



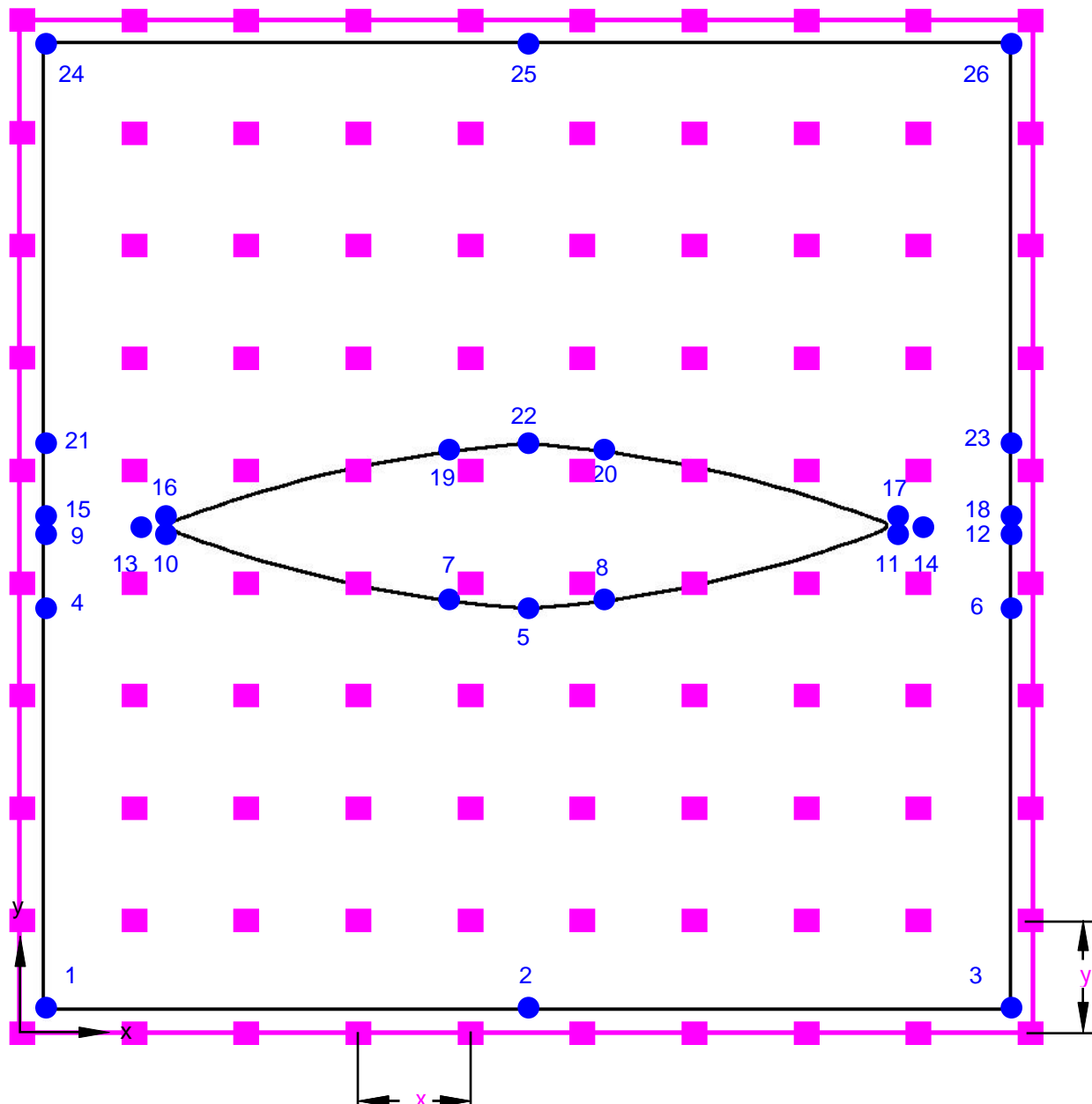
extent of contour map

(a). Electron micrograph of individual z-pin with surrounding laminate [13]



(b). Contour map of fiber alignment angle [13]

Figure 7: Measurement of fiber misalignment angle



\*GEOMETRICAL POINTS

|     |        |        |
|-----|--------|--------|
| 26  |        |        |
| 1   | 0.00   | 0.00   |
| 2   | 125.00 | 0.00   |
| 3   | 250.00 | 0.00   |
| 4   | 0.00   | 103.60 |
| 5   | 125.00 | 103.60 |
| 6   | 250.00 | 103.60 |
| 7   | 110.00 | 105.00 |
| ... |        |        |
| ... |        |        |
| 26  | 250.00 | 250.00 |

\*MISALIGNMENT TYPE

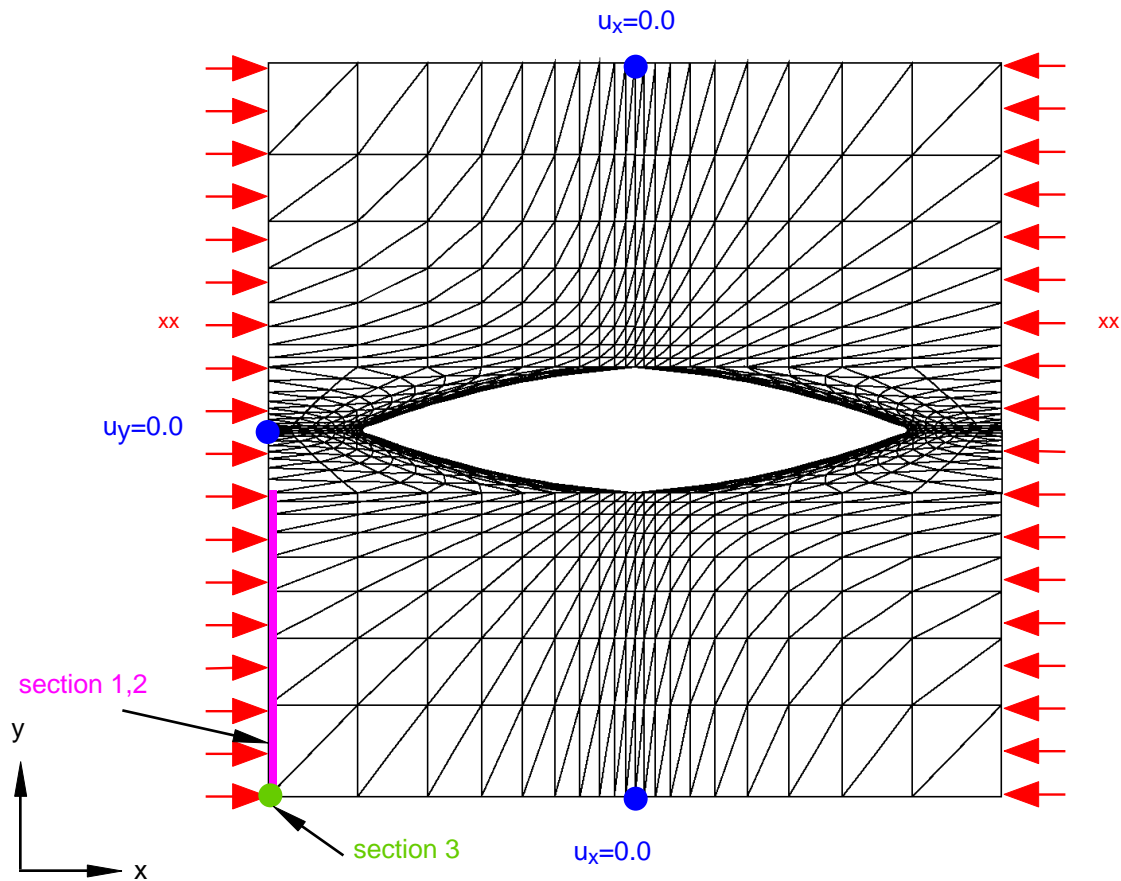
2

\*MISALIGNMENT

10,10,26.0,26.0

Figure 8: Outline of unit cell with geometry points and fiber misalignment data points





**\*STRESS LOADING**

```

10, 1
1, 1, 4, 1, 1
-1000, 0, 0, 0, 0
4, 4, 9, 1, 1
-1000, 0, 0, 0, 0
9, 9, 15, 1, 1
-1000, 0, 0, 0, 0
15, 15, 21, 1, 1
-1000, 0, 0, 0, 0
21, 21, 24, 1, 1
-1000, 0, 0, 0, 0
3, 3, 6, 1, 1
-1000, 0, 0, 0, 0
6, 6, 12, 1, 1
-1000, 0, 0, 0, 0
12, 12, 18, 1, 1
-1000, 0, 0, 0, 0
18, 18, 23, 1, 1
-1000, 0, 0, 0, 0
23, 23, 26, 1, 1
-1000, 0, 0, 0, 0

```

**\*TYPE 1 CONSTRAINTS**

```

3
1, 1, 2, 21, 1, 1, 0
9, 9, 15, 4, 4, 2, 0
24, 24, 25, 21, 1, 1, 0

```

**\*OUTPUT LIST**

```

3
1, 1, 4, 1, 1
1, 1, 4, 1, 1
1, 1, 4, 1, 17

```

**Figure 9: Finite element model subjected to axial compression loading**

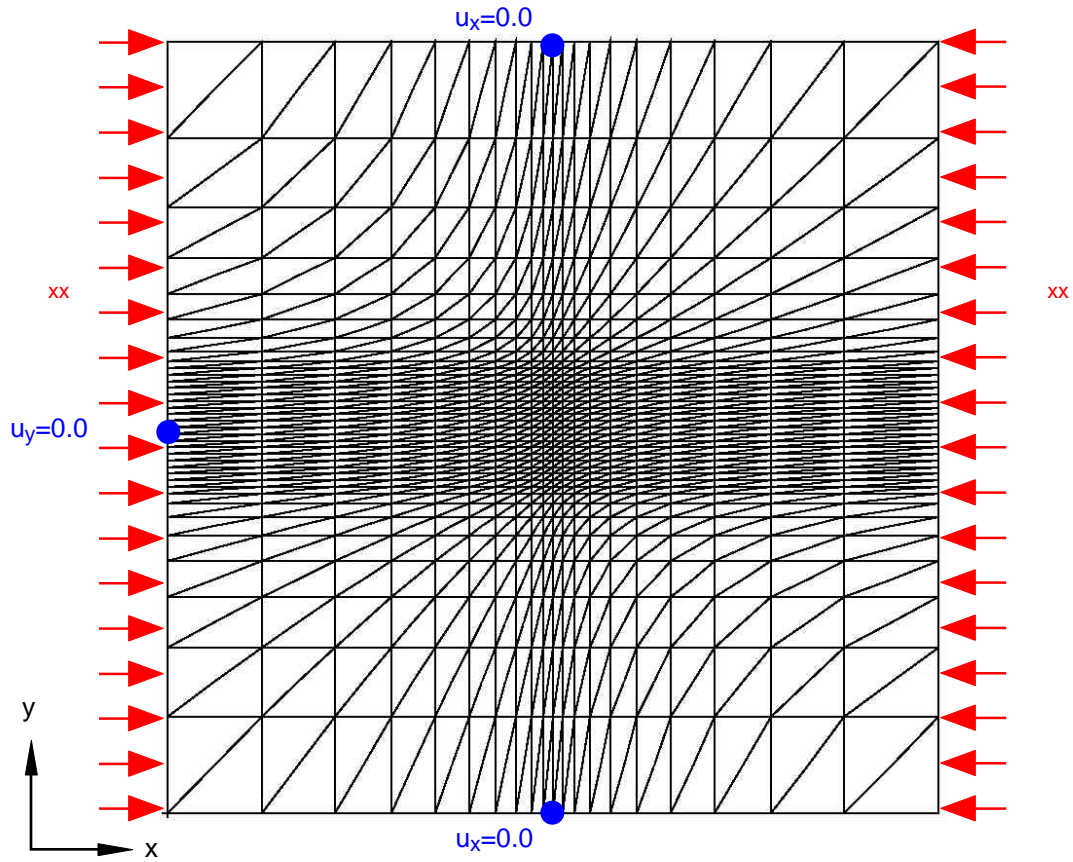
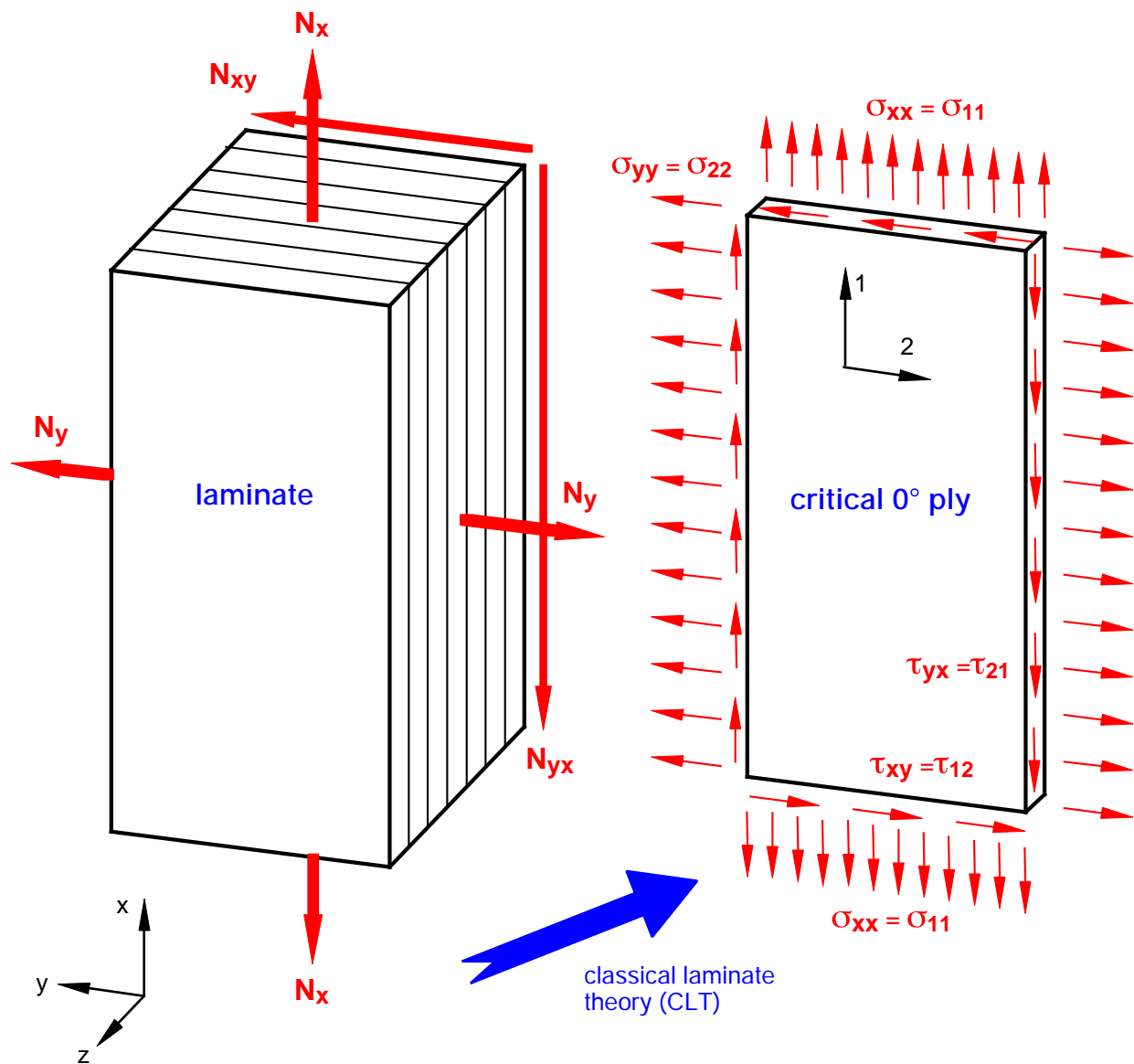


Figure 10: Mesh with axial compression load and boundary conditions for UD Ply without Z-Pin



two load cases  
 $N_x = 1000 \text{ lbs/in}$   
 $N_x = 1000 \text{ lbs/in}, N_{xy} = 1000 \text{ lbs/in}$

three laminates  
 $[0/90]_s$   
 $[0/\pm 45]_s$   
 $[0/45/-45/90]_s$

four critical cases identified  
 $\sigma_{xx}$  (compression) only  
 $\sigma_{xx}$  (compression),  $\sigma_{yy}$  (tension) = 2%  $\sigma_{xx}$   
 $\sigma_{xx}$  (compression),  $\tau_{xy} = \tau_{yx} = 10\% \sigma_{xx}$   
 $\sigma_{xx}$  (compression),  $\tau_{xy} = \tau_{yx} = 10\% \sigma_{xx}$ ,  $\sigma_{yy}$  (tension) = 2%  $\sigma_{xx}$

Figure 11: Calculation of ply stresses using classical laminate theory

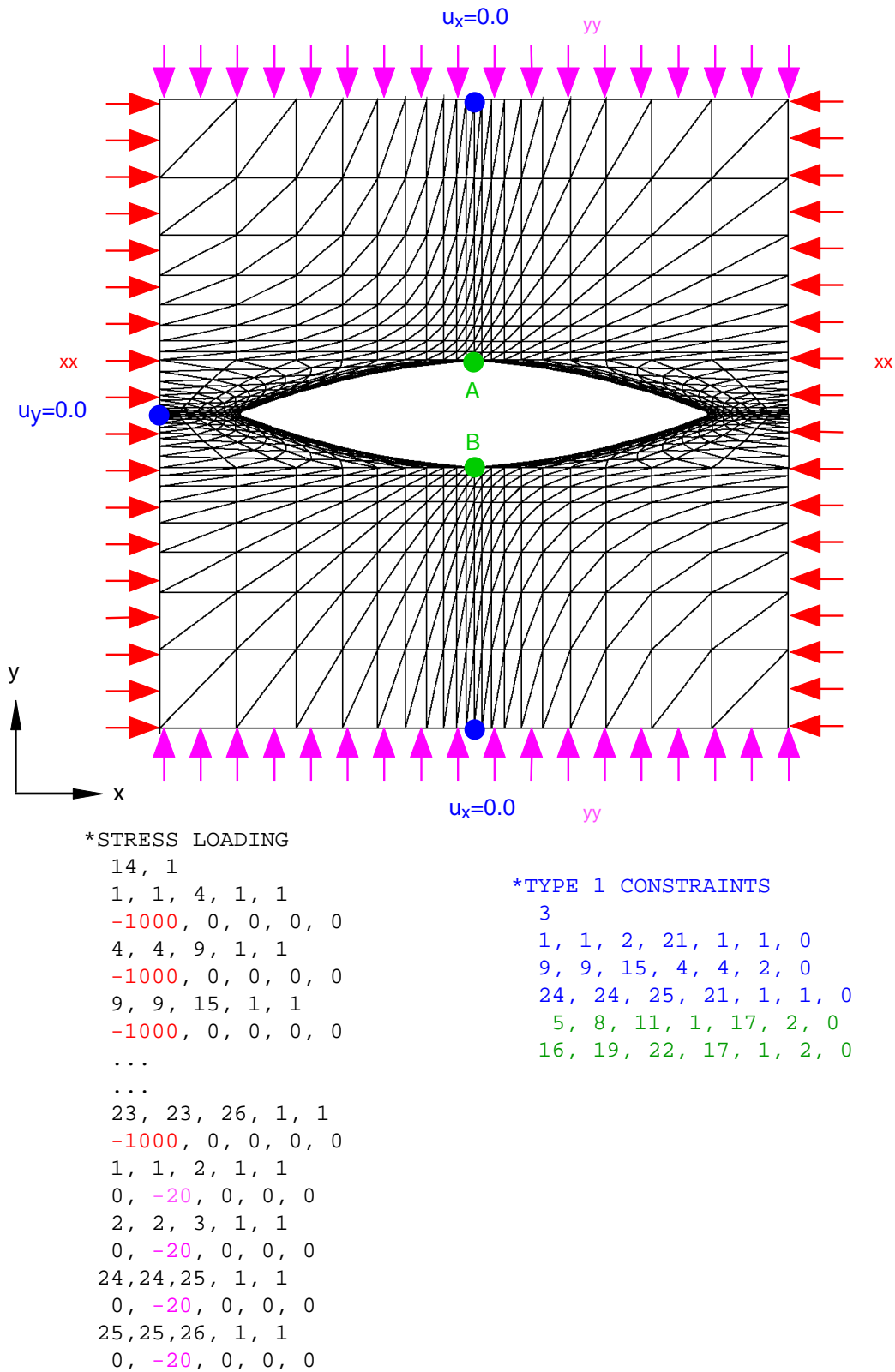


Figure 12: Finite element model subjected to axial and transverse compression loading

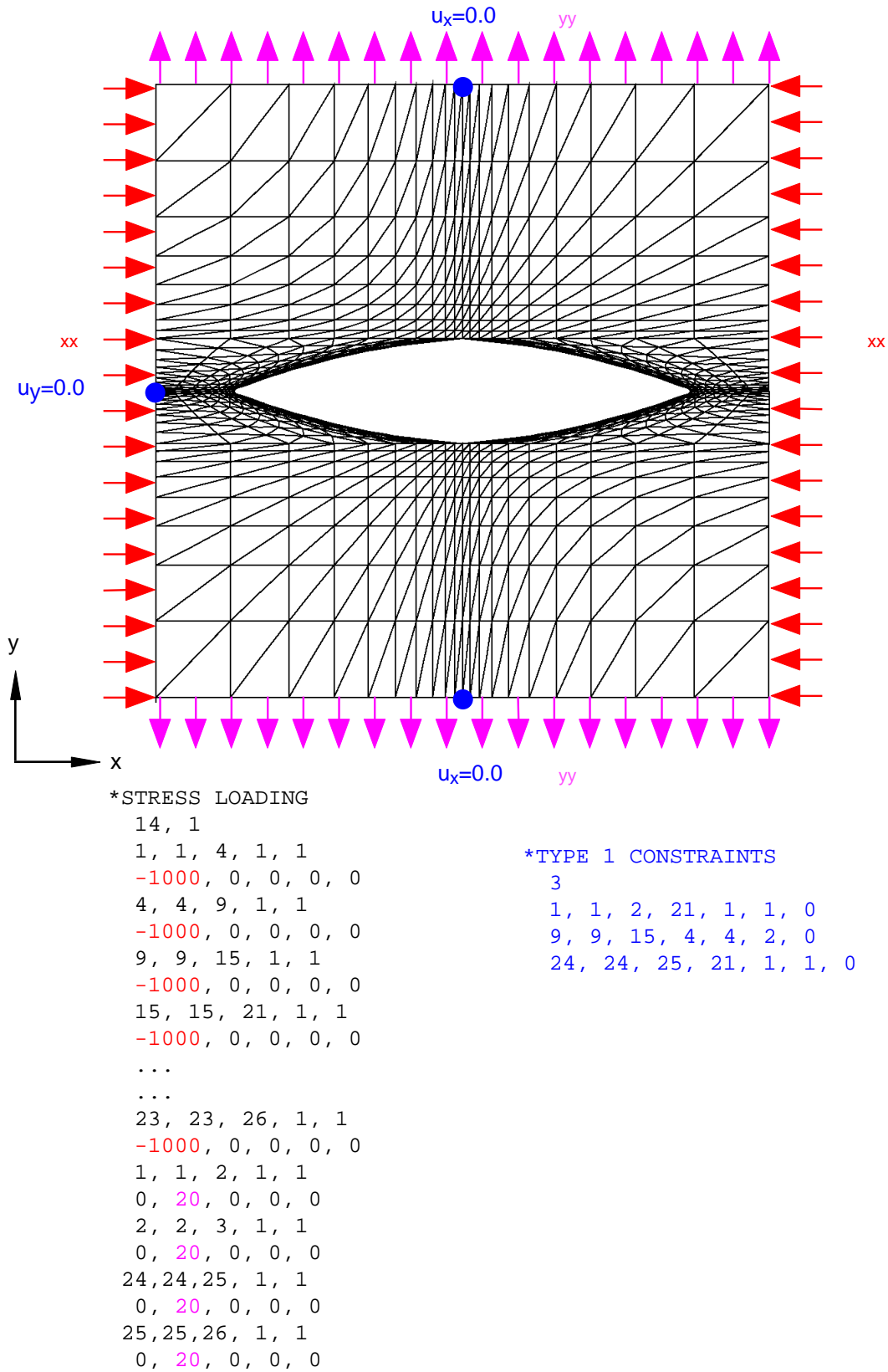
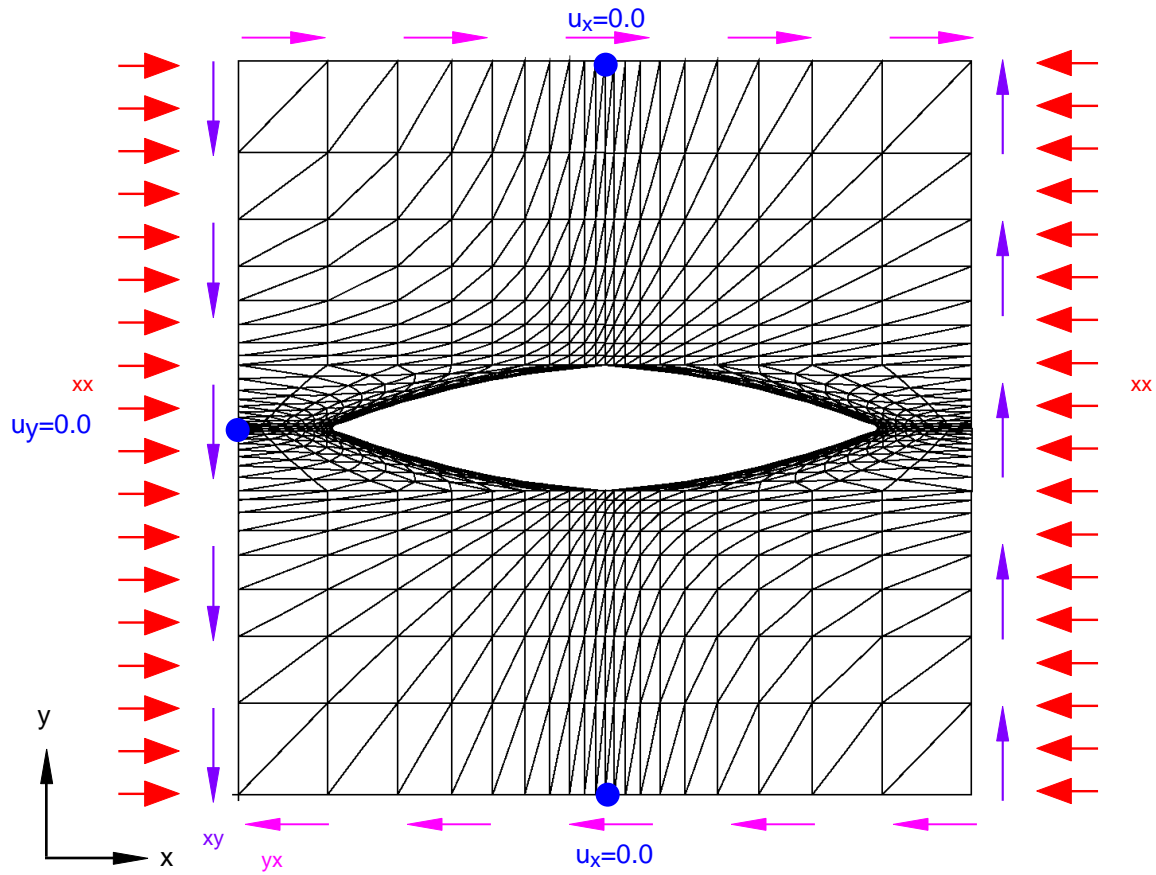


Figure 13: Finite element model subjected to axial compression and transverse tension loading



\*STRESS LOADING

```

14, 1
1, 1, 4, 1, 1
-1000, 0, -100, 0, 0
4, 4, 9, 1, 1
-1000, 0, -100, 0, 0
9, 9, 15, 1, 1
-1000, 0, -100, 0, 0
15, 15, 21, 1, 1
-1000, 0, -100, 0, 0
...
...
23, 23, 26, 1, 1
-1000, 0, -100, 0, 0
1, 1, 2, 1, 1
0, 0, 0, -100, 0
2, 2, 3, 1, 1
0, 0, 0, -100, 0
24, 24, 25, 1, 1
0, 0, 0, -100, 0
25, 25, 26, 1, 1
0, 0, 0, -100, 0

```

\*TYPE 1 CONSTRAINTS

```

3
1, 1, 2, 21, 1, 1, 0
9, 9, 15, 4, 4, 2, 0
24, 24, 25, 21, 1, 1, 0

```

Figure 14: Finite element model subjected to axial compression and shear loading

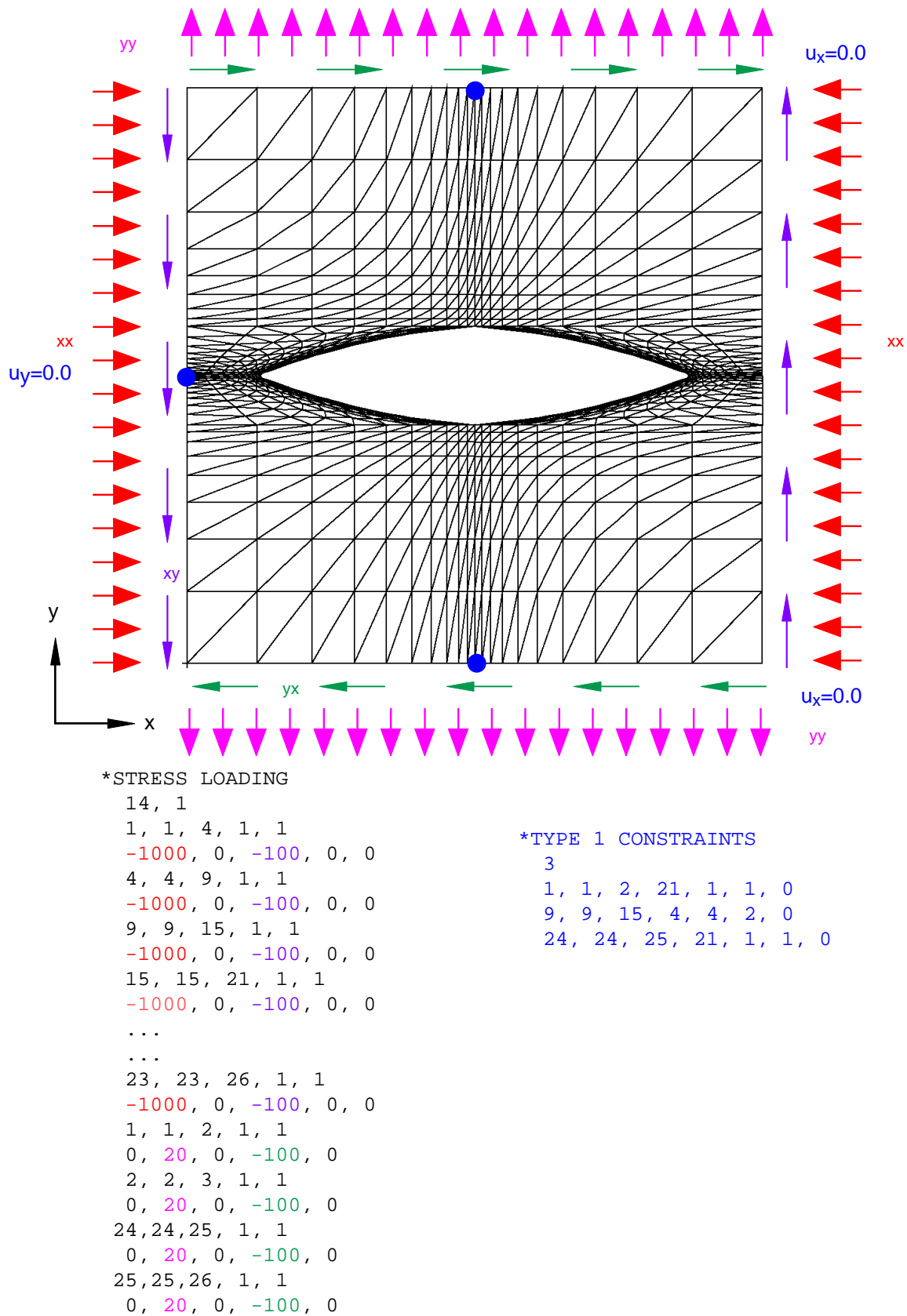
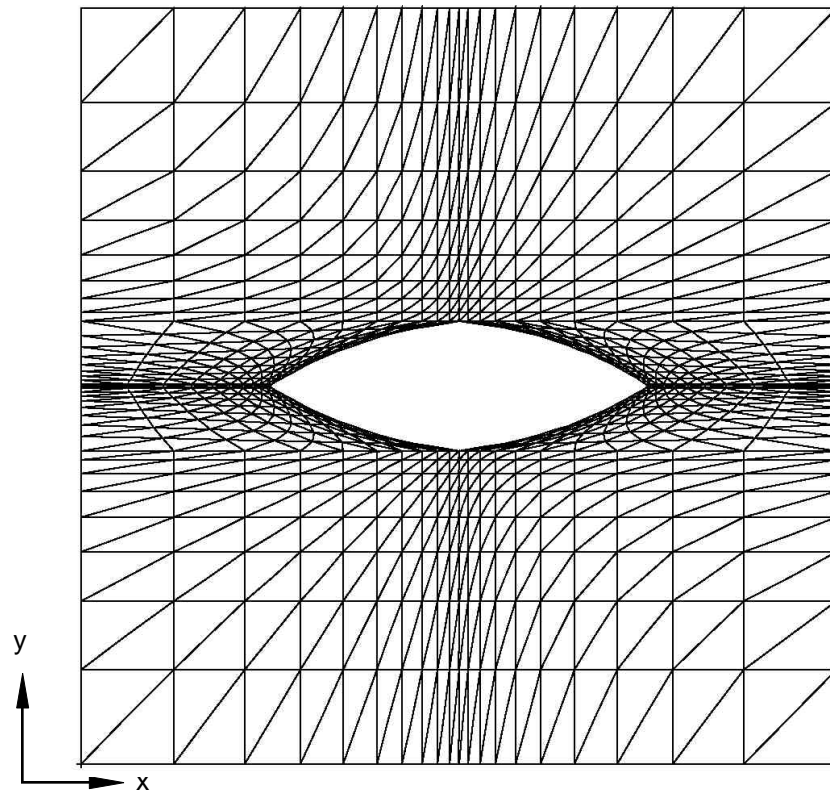


Figure 15: Finite element model subjected to compression, transverse tension and shear loading



laminate with  
2% carbon z-pins  
 $C = 0.868 \text{ mm}$   
 $D_z = 0.280 \text{ mm}$   
 $D'_z = 0.3 \text{ mm}$   
 $H_z = 1.7526 \text{ mm}$   
 $L_z = 1.7526 \text{ mm}$   
 $d = 0.0051 \text{ mm}$

$C/d = 170.2$   
 $D'_z/d = 58.8$   
 $H_z/d = 343.65$   
 $L_z/d = 343.65$

\*GEOMETRICAL POINTS

|    |        |        |
|----|--------|--------|
| 26 |        |        |
| 1  | 0.00   | 0.00   |
| 2  | 171.85 | 0.00   |
| 3  | 343.7  | 0.00   |
| 4  | 0.00   | 142.45 |
| 5  | 171.85 | 142.45 |
| 6  | 343.7  | 142.45 |
| 7  | 158.25 | 144.35 |
| 8  | 185.75 | 144.35 |
| 9  | 0.00   | 170.85 |
| 10 | 86.75  | 170.85 |
| 11 | 256.95 | 170.85 |
| 12 | 343.7  | 170.85 |
| 13 | 86.05  | 171.85 |
| 14 | 257.65 | 171.85 |
| 15 | 0.00   | 172.85 |
| 16 | 86.75  | 172.85 |
| 17 | 256.95 | 172.85 |
| 18 | 343.7  | 172.85 |
| 19 | 158.25 | 199.35 |
| 20 | 187.75 | 199.35 |
| 21 | 0.00   | 201.25 |
| 22 | 171.85 | 201.25 |
| 23 | 343.7  | 201.25 |
| 24 | 0.00   | 343.7  |
| 25 | 171.85 | 343.7  |
| 26 | 343.7  | 343.7  |

\*MISALIGNMENT TYPE

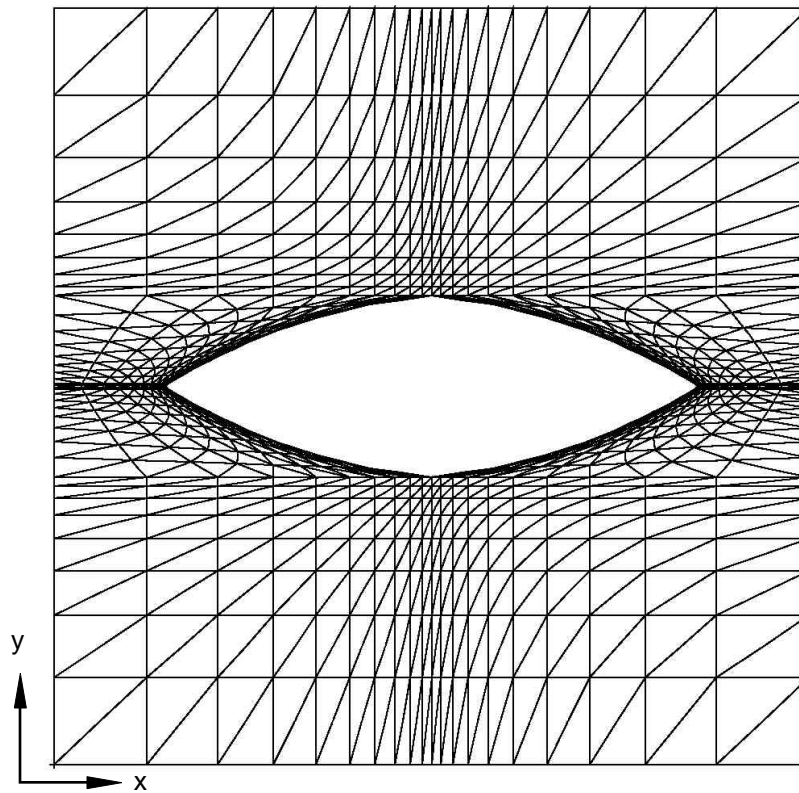
2

\*MISALIGNMENT

74,74,4.7,4.7

Figure 16:Finite element mesh of unit cell for small z-pin with 2% areal density





laminate with  
4% carbon z-pins  
 $C = 0.868 \text{ mm}$   
 $D_z = 0.280 \text{ mm}$   
 $D'_z = 0.3 \text{ mm}$   
 $H_z = 1.2446 \text{ mm}$   
 $L_z = 1.2446 \text{ mm}$   
 $d = 0.0051 \text{ mm}$

$C / d = 170.2$   
 $D'_z / d = 58.8$   
 $H_z / d = 244$   
 $L_z / d = 244$

\*GEOMETRICAL POINTS

|    |        |        |
|----|--------|--------|
| 26 |        |        |
| 1  | 0.00   | 0.00   |
| 2  | 122.00 | 0.00   |
| 3  | 244.00 | 0.00   |
| 4  | 0.00   | 92.60  |
| 5  | 122.00 | 92.60  |
| 6  | 244.00 | 92.60  |
| 7  | 108.10 | 94.50  |
| 8  | 135.90 | 94.50  |
| 9  | 0.00   | 121.00 |
| 10 | 36.90  | 121.00 |
| 11 | 207.10 | 121.00 |
| 12 | 244.00 | 121.00 |
| 13 | 36.20  | 122.00 |
| 14 | 207.80 | 122.00 |
| 15 | 0.00   | 123.00 |
| 16 | 36.90  | 123.00 |
| 17 | 207.10 | 123.00 |
| 18 | 244.00 | 123.00 |
| 19 | 108.10 | 149.50 |
| 20 | 135.90 | 149.50 |
| 21 | 0.00   | 151.40 |
| 22 | 122.00 | 151.40 |
| 23 | 244.00 | 151.40 |
| 24 | 0.00   | 244.00 |
| 25 | 122.00 | 244.00 |
| 26 | 244.00 | 244.00 |

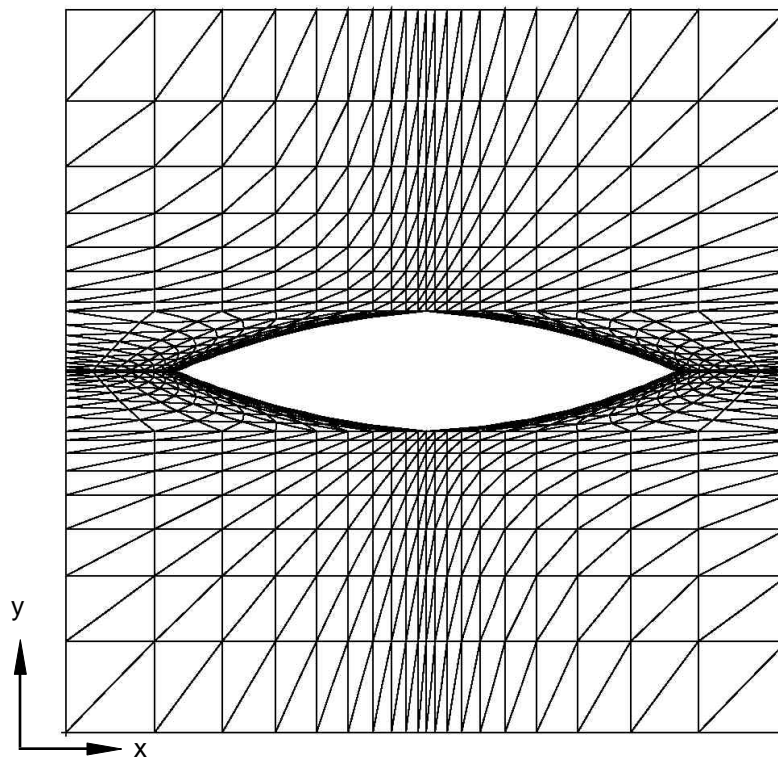
\*MISALIGNMENT TYPE

2

\*MISALIGNMENT

74,74,3.4,3.4

Figure 17: Finite element mesh of unit cell for small z-pin with 4% areal density



laminate with  
2% carbon z-pins  
 $C = 2.1844 \text{ mm}$   
 $D_z = 0.508 \text{ mm}$   
 $D'_z = 0.528 \text{ mm}$   
 $H_z = 3.175 \text{ mm}$   
 $L_z = 3.175 \text{ mm}$   
 $d = 0.0051 \text{ mm}$

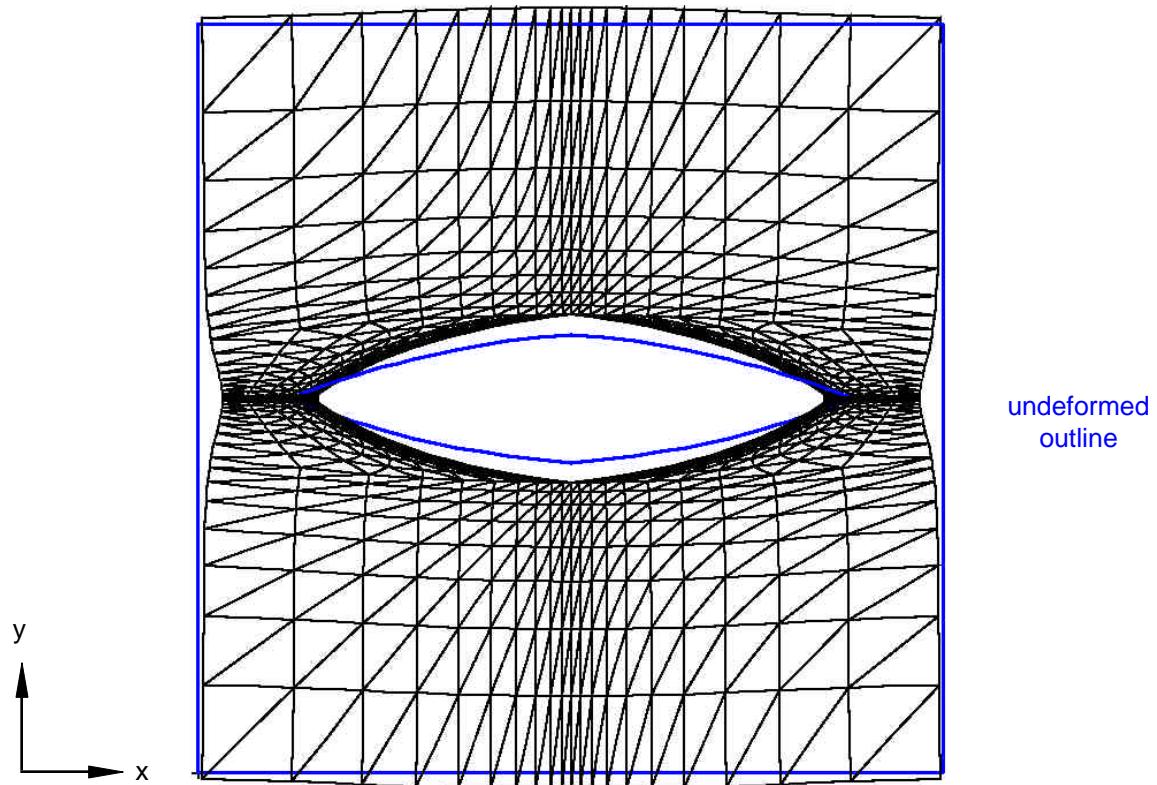
$C / d = 428.31$   
 $D'_z / d = 103.53$   
 $H_z / d = 622.55$   
 $L_z / d = 622.55$

\*GEOMETRICAL POINTS

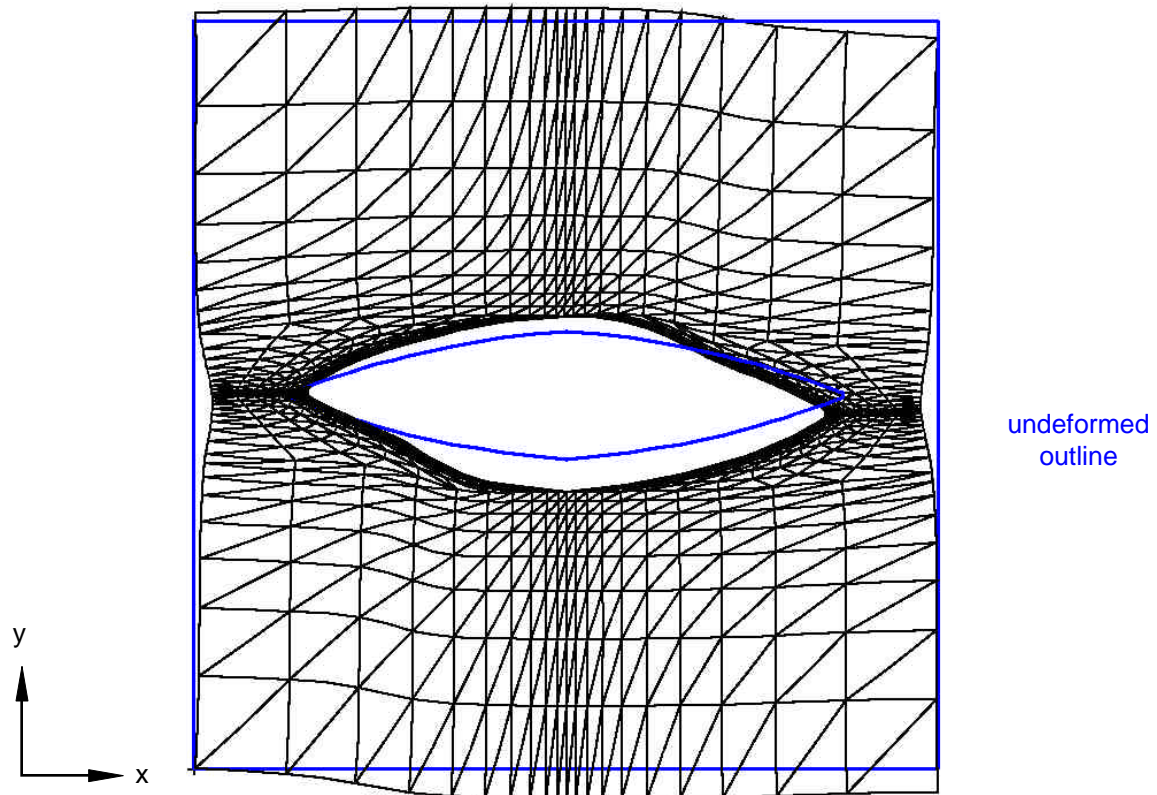
|    |         |         |
|----|---------|---------|
| 26 |         |         |
| 1  | 0.00    | 0.00    |
| 2  | 311.275 | 0.00    |
| 3  | 622.55  | 0.00    |
| 4  | 0.00    | 259.51  |
| 5  | 311.275 | 259.51  |
| 6  | 622.55  | 259.51  |
| 7  | 276.275 | 262.91  |
| 8  | 346.275 | 261.91  |
| 9  | 0.00    | 310.275 |
| 10 | 97.12   | 310.275 |
| 11 | 525.43  | 310.275 |
| 12 | 622.55  | 310.275 |
| 13 | 96.12   | 311.275 |
| 14 | 526.43  | 311.275 |
| 15 | 0.00    | 312.275 |
| 16 | 97.12   | 312.275 |
| 17 | 525.43  | 312.275 |
| 18 | 622.55  | 312.275 |
| 19 | 276.275 | 359.64  |
| 20 | 346.275 | 359.64  |
| 21 | 0.00    | 363.04  |
| 22 | 311.275 | 363.04  |
| 23 | 622.55  | 363.04  |
| 24 | 0.00    | 622.55  |
| 25 | 311.275 | 622.55  |
| 26 | 622.55  | 622.55  |

\*MISALIGNMENT TYPE  
2  
\*MISALIGNMENT  
74,74,8.6,8.6

Figure 18:Finite element mesh of unit cell for large z-pin with 2% areal density



a. Deformed mesh after 10 load increments



b. Deformed mesh after 1000 load increments

Figure 19: Analysis of unit cell of UD Ply with Z-Pin subjected to axial compression



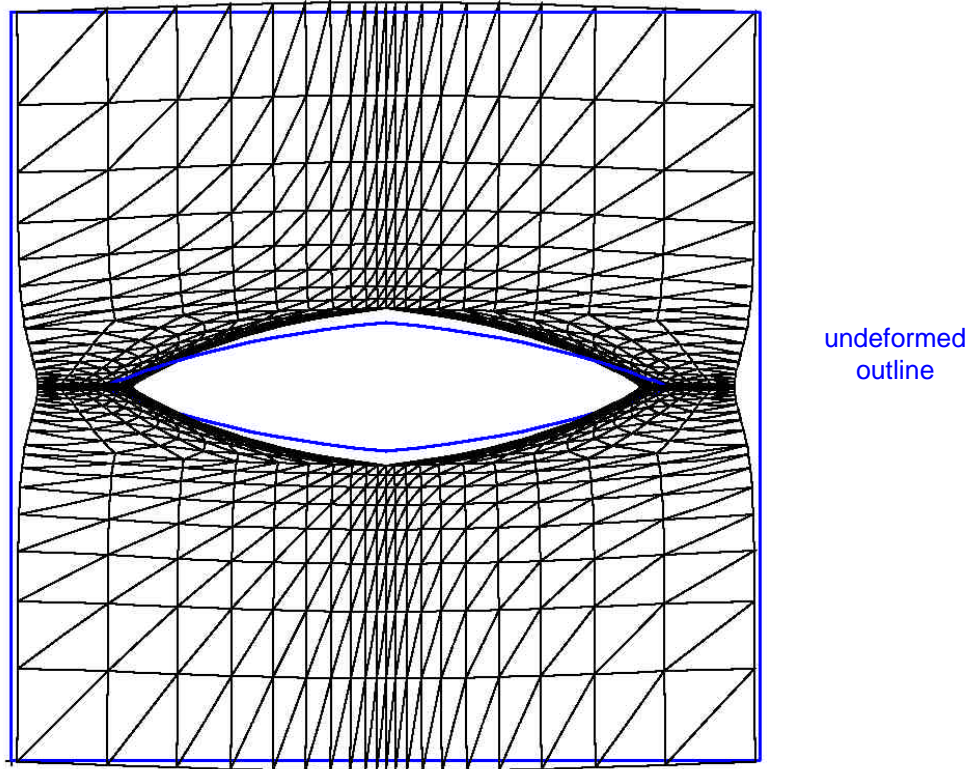


Figure 20: Deformed finite element model of unit cell subjected to axial and transverse compression loading

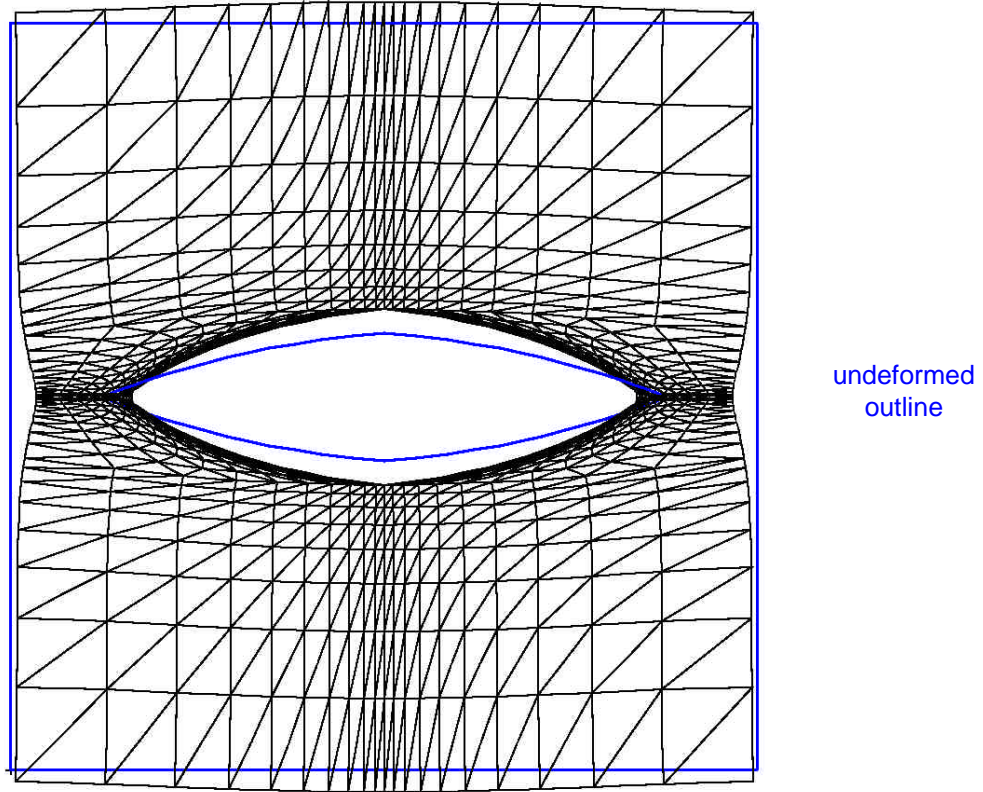
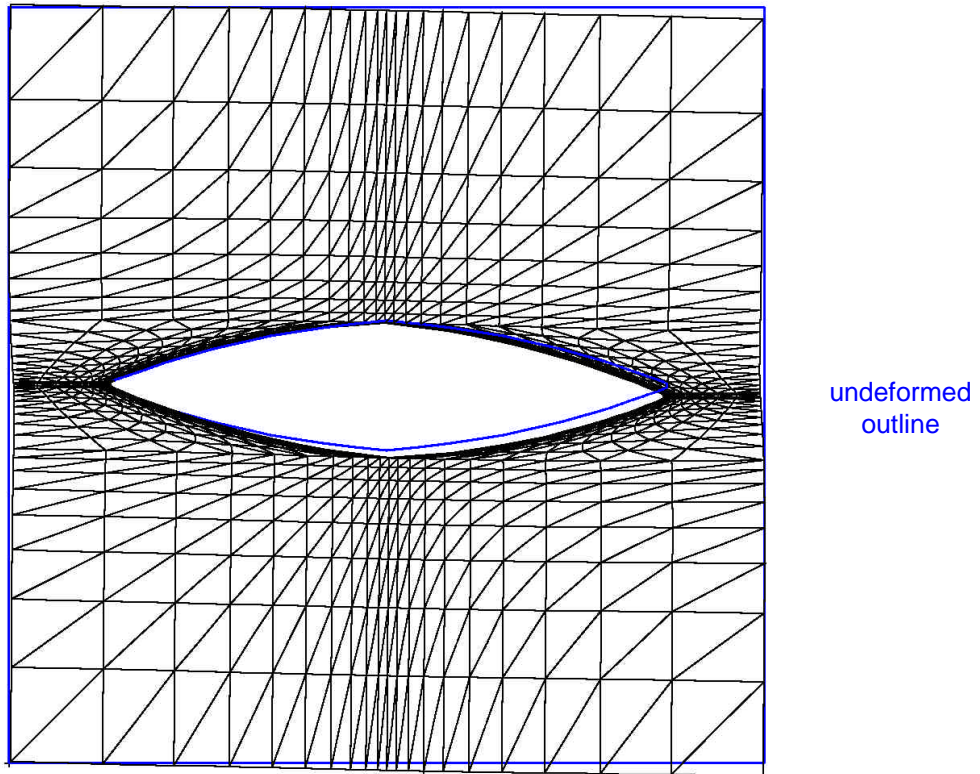
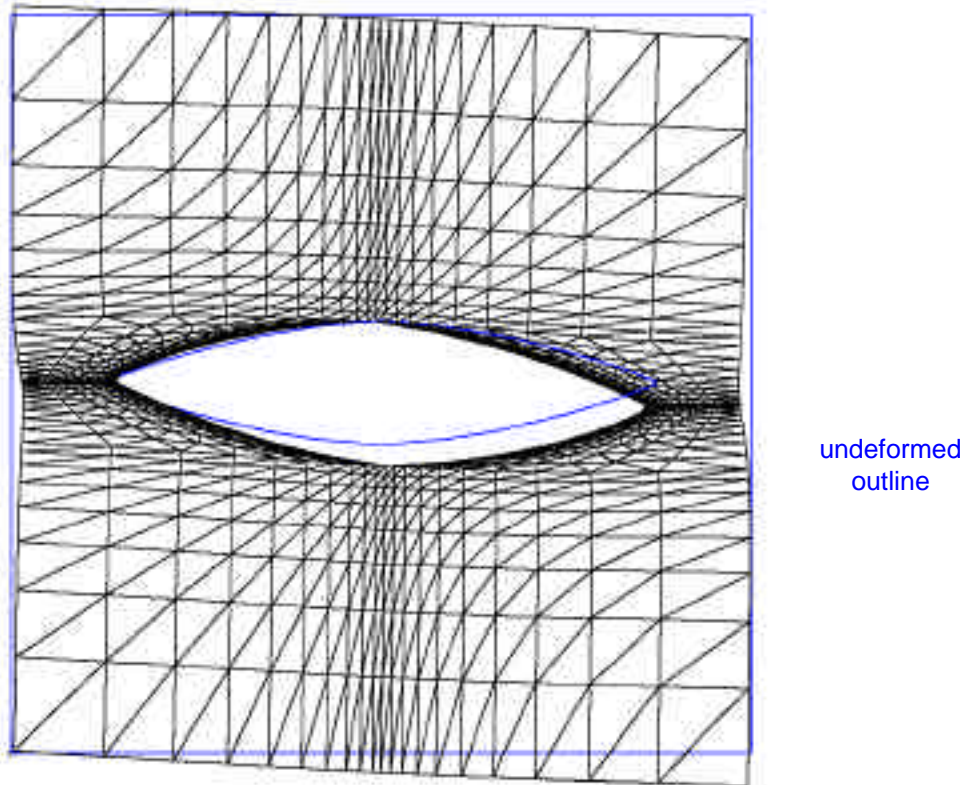


Figure 21: Deformed finite element model of unit cell subjected to axial compression and transverse tension loading



**Figure 22: Deformed finite element model of unit cell subjected to axial compression and shear loading**



**Figure 23: Deformed finite element model of unit cell subjected to axial compression, transverse tension and shear loading**

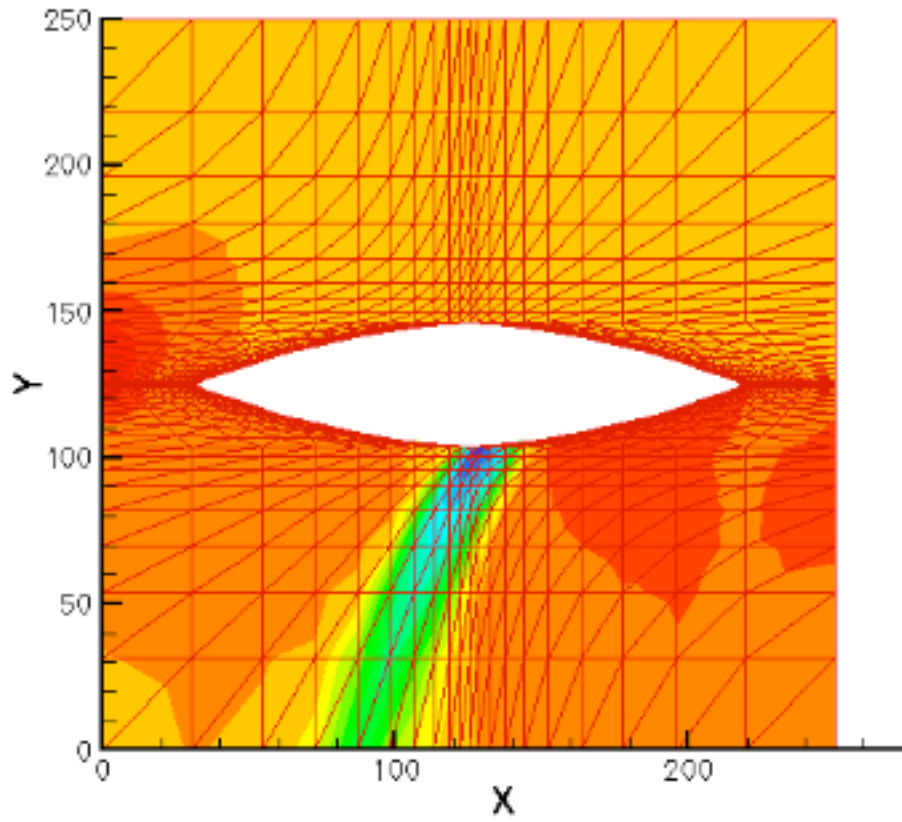


Figure 24: Contour plot of fiber misalignment angle  $\phi$

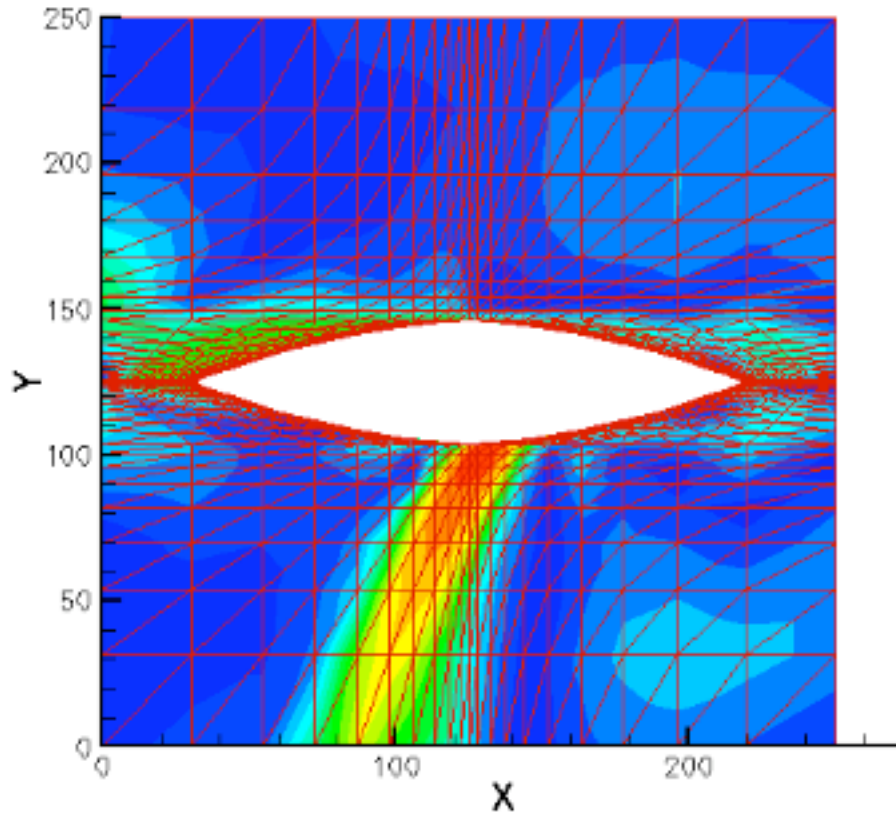


Figure 25: Contour plot of shear stress  $\tau_e$



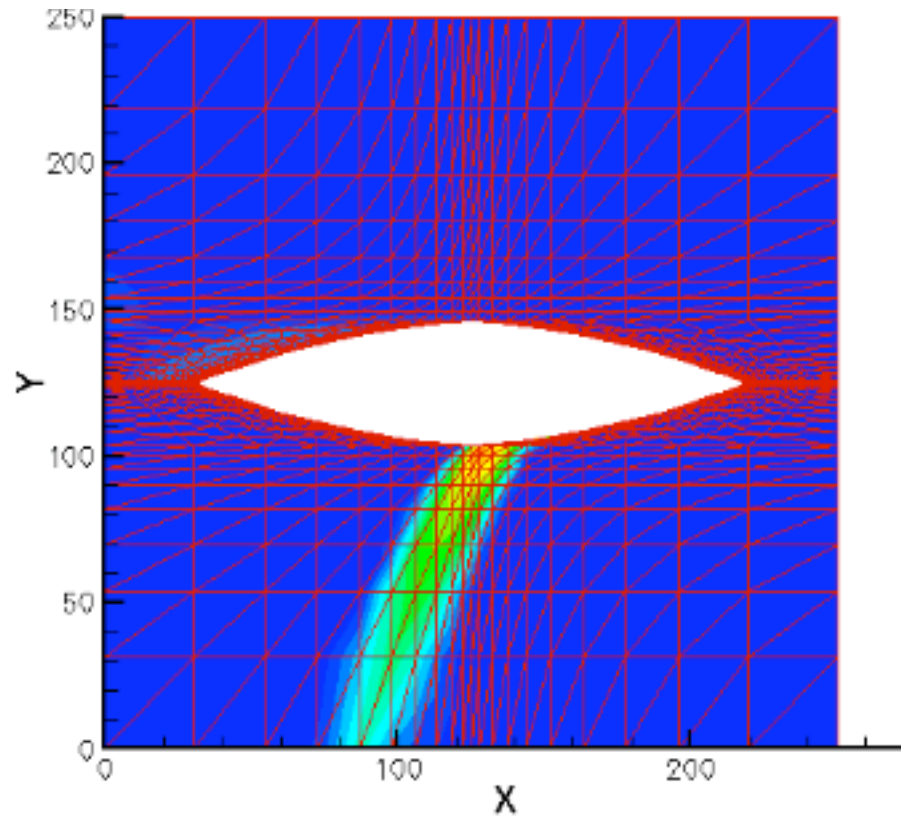


Figure 26: Contour plot of shear strain  $\gamma_p$

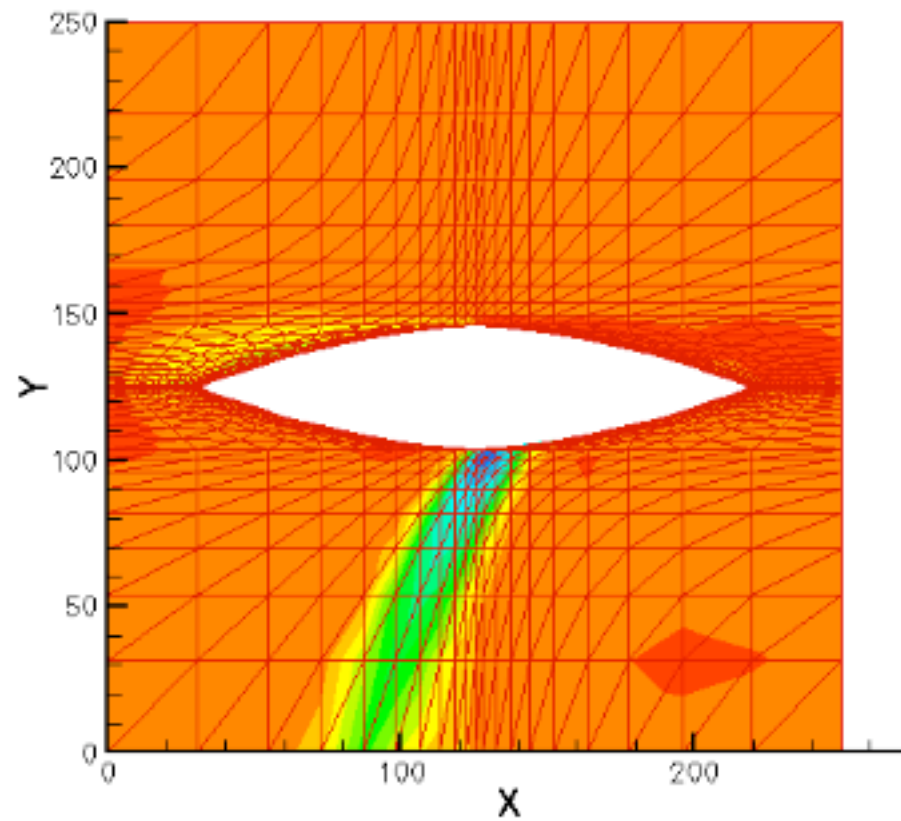


Figure 27: Contour plot of shear strain  $\gamma_s$

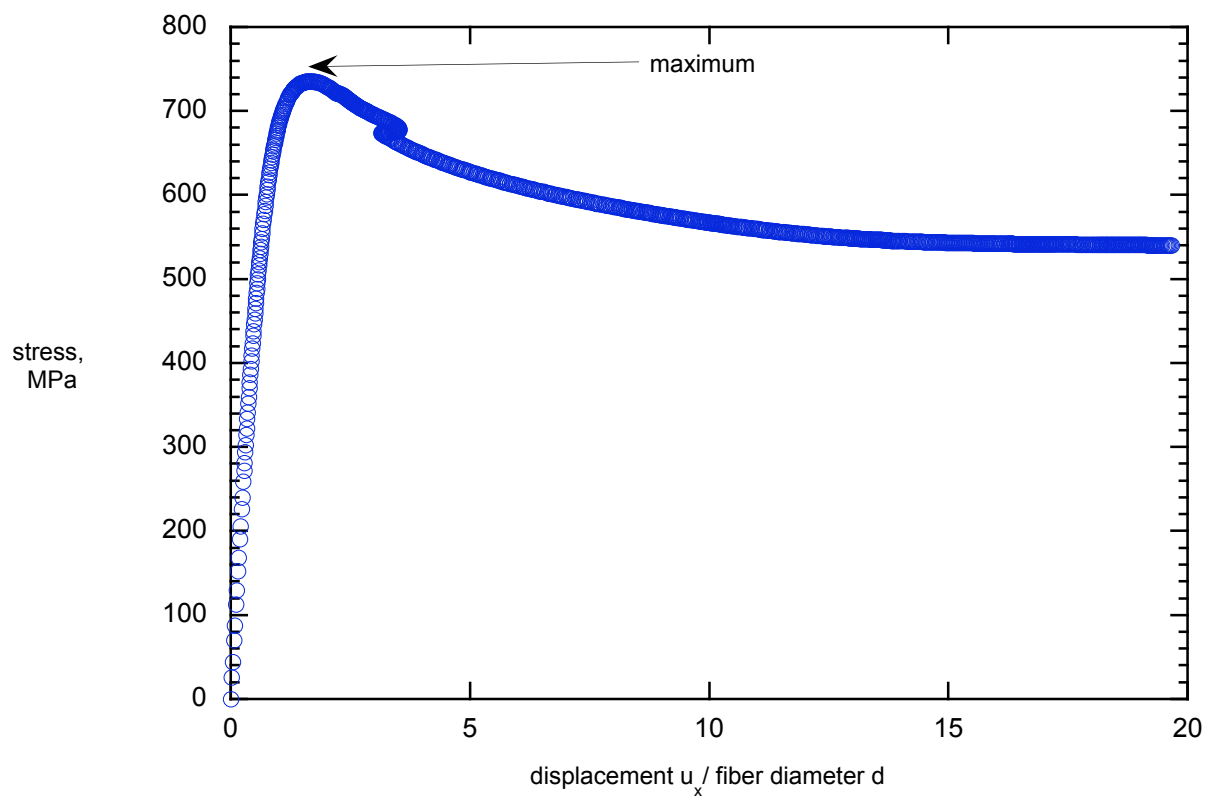


Figure 28. Stress-displacement plot

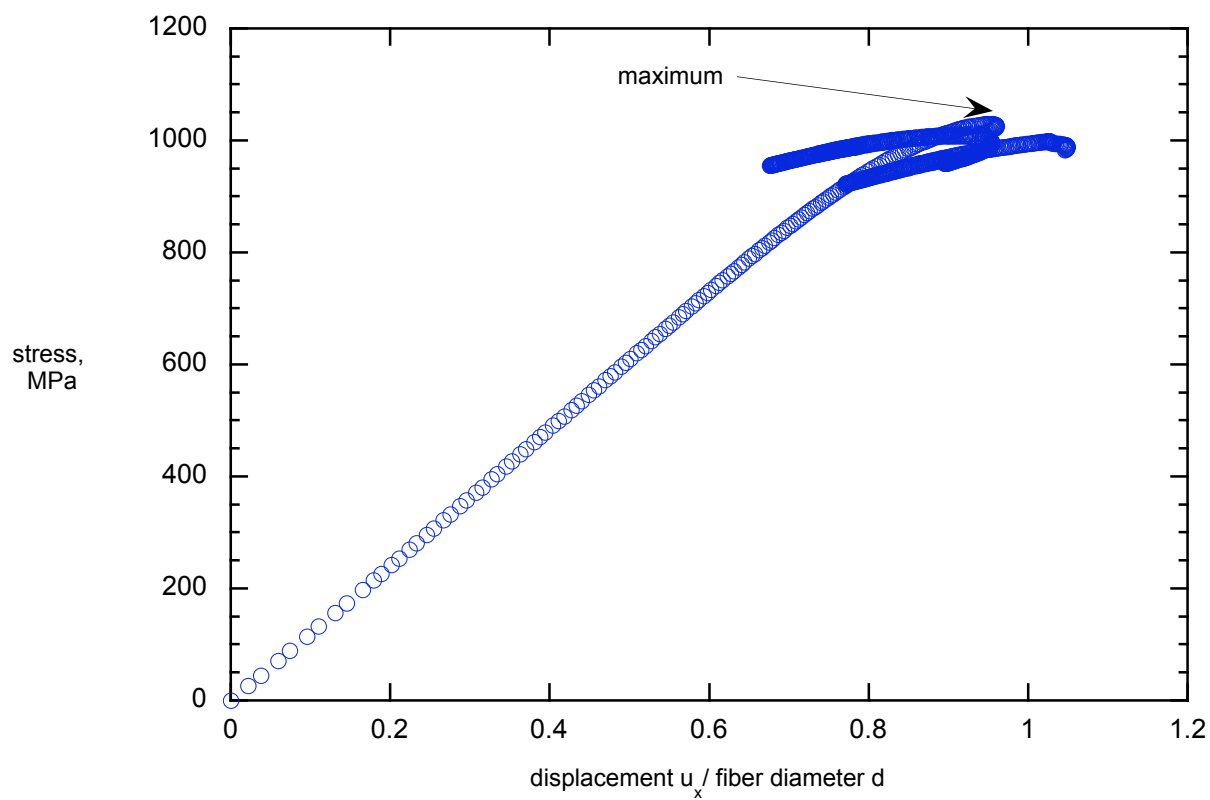


Figure 29. Stress-displacement plot



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|   |                    |                       |                                   | <b>11. SPONSORING/MONITORING REPORT NUMBER</b>   |  |
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| <b>15. SUBJECT TERMS</b>  |                    |                       |                                   |  |  |
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